ALTERNATING-CURRENT MACHINERY

BY THE SAME AUTHOR

Alternating-Current Problems

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ALTERNATING-CURRENT MACHINERY

A Text-book on the Theory and Performance of Generators and Motors

BY

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BANGALORE

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PREFACE

THE purpose of this book is to provide a text-book on the principles of alternating-current machinery which thoroughly treats the subject and which may at the same time be adapted to almost any type of course. It is best suited for students who already have a good training on the principles of alternating-current circuits.

The book is so constructed that the best engineering students should have little, but it is hoped some, difficulties in understanding certain parts. The average student will have the usual difficulties, but a book that leaves nothing for the students or for the instructor to do is hardly a text-book. It is felt that a book containing a few hundred pages to think about is a better training book than one containing a thousand pages of information readily understood by all. It is hoped that this is more of a training book than a mere information book.

Throughout the book emphasis has been placed on the analysis of the performance of the machines. Lengthy descriptions of the construction of machines have been avoided for it was felt that such information should be obtained from technical journals or technical literature furnished by the manufacturing companies.

Since there are few repetitions in this book it is desirable to read it in the order in which it is written. Explanations are given for everything, but one who reads it may sometimes be required to refer back to previous articles.

The problems at the end of nearly every chapter are designed with the idea of illustrating the principles of operation of alternating-current machinery. The problems are not ridiculously simple which unfortunately is the case in many text-books.

The problems in Chapter XX which are to be solved in the machinery laboratory may be adapted to any equipment. The problems are so designed that failure to obtain a solution of a problem in a given laboratory period results in the reassignment of the same problem on the following period. It is hoped that the student is required to locate and remedy any difficulties he experiences in solving these problems. It is also hoped that the student solves none of these problems as a mere "stunt" to be accomplished at as great a speed as possible.

The articles on armature reaction, parallel operation of alternators, single-phase motors, and the phase balancer contain abstracts taken from the original notes on alternating-current machinery by Dr. Harold Pender.

The material in this book in note form has been used in the author's classes for several years.

R. E. Brown

Philadelphia, Pa.,
August 29, 1927.

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ALTERNATING-CURRENT MACHINERY

CHAPTER I

ALTERNATING-CURRENT THEORY

1. Introduction.—The treatment of alternating-current machinery given in this book assumes a knowledge of the elementary principles of alternating-currents. For convenience of reference, a summary of the elementary principles required in a course on machinery ~~are~~ given in this chapter. (For a thorough treatment of the principles of alternating-currents, the reader is referred to Dr. Pender's second volume of " Electricity and Magnetism for Engineers " McGraw-Hill Book Co.).

2. Alternating-current Waves.—An alternating-current varies continuously with time from a maximum value in one direction to a maximum value in the opposite direction, and back again to the same maximum value in the first direction.

In Fig. 1, the successive instantaneous values of an alternating-current are plotted as ordinates against time as abscisses. The wave shown in this diagram is symmetrical; that is, the top half and the bottom half of the wave form are identical.

3. General Definitions.—The following definitions are stated in terms of current, but are equally applicable to any other alternating quantity.

Period.—The period of an alternating-current is the time required for the current to pass through a complete cycle of positive and negative values. In Fig. 1, a complete period is represented by T seconds, 2π radians, or 360 degrees.

Frequency.—The frequency of an alternating-current is the number of complete periods in a second. The frequency f is expressed in cycles per second.

The relation between the frequency and the period is

$$f=\frac{1}{T} \tag{1}$$

Phase.—The abscissa of any point on a wave form is called the "phase" of the ordinate at this point, referred to the origin of the system of co-ordinates employed. The phase of the ordinate Q–P in Fig. 1 is the distance O–Q.

Instantaneous value.—The instantaneous value of an alternating-current is the value of the current at any instant.

Maximum value.—The maximum value of an alternating-current is the greatest instantaneous value during a complete cycle.

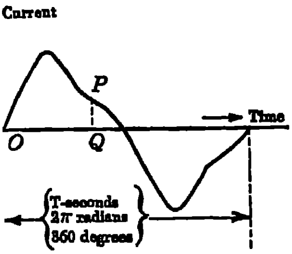


FIG. 1.—Alternating-current Wave

Average value.—The average value of an alternating-current over a complete cycle is zero. The term average value is used, however, to denote the average ordinate of a portion of the wave form (usually between two zero values).

R.M.S. value.—The r.m.s. value of an alternating-current is the square root of the mean of the squares of the instantaneous values over a complete cycle. The mathematical expression for the r.m.s. value of the current is

$$\text{R.M.S. value}=\sqrt{\frac{1}{T}\int_0^T i^2dt} \tag{2}$$

The r.m.s. value of the current in a circuit is determined by the indication of an ammeter. Similarly, the r.m.s. value of a voltage is determined by a voltmeter.

Form factor.—The ratio of the r.m.s. value of an alternating-current to its average value is called the form factor.

$$\text{Form factor}=\frac{I_{\text{r.m.s.}}}{I_{\text{av.}}} \tag{3}$$

Crest factor.—The ratio of the maximum value of an alternating-current to its r.m.s. value is called the crest factor.

$$\text{Crest factor}=\frac{I_{\text{max.}}}{I_{\text{r.m.s.}}} \tag{4}$$

4. Sine-wave Quantities.—Practically all modern a-c. generators are so designed that they generate electromotive forces which, to a close degree of approximation, vary sinusoidally with time. The usual circuit calculations having to do with steady state conditions in a-c. circuits are therefore based on the assumption that the voltage applied to the circuit is a sinusoidal quantity.

The wave shape of the current delivered by a generator depends on the wave shape of the voltage generated and on the value and variation throughout each cycle, of the equivalent resistance, equivalent inductance, and equivalent electrostatic capacity of the complete circuit to which the generator is connected. If the equivalent resistance, inductance, and capacity are absolutely constant, a sinusoidal voltage produces a sinusoidal current. This is the condition existing approximately in a-c. machinery circuits; especially is this true when the machines are operated at rated load.

A sinusoidal electromotive force of the form

$$e = E_{max} \sin \omega t \qquad (5)$$

acting on a circuit of constant so-called constants produces a current of the form

$$i = I_{max.} \sin (\omega t \pm \theta) \qquad (6)$$

where

e = instantaneous value of the e.m.f.;

E_{max} = maximum value of the e.m.f.;

i = instantaneous value of current;

$I_{max.}$ = maximum value of current;

$\omega t = (2\pi f)t$ radians;

t = the time in seconds referred to the first ascending zero value of the electromotive force;

and θ = the phase difference in radians between the first ascending zero values of the e.m.f. and the current. The value and sign of this phase difference depends on the kind of circuit.

5. Relation Between Maximum and R.M.S. Values of Sinusoidal Quantities.—If a direct-current of value I flows through a resistance r, the average power loss in the resistance over any interval of time is rI^2 watts. Similarly, if an alternating-current of the form $i=I_{\text{max}}\sin(\omega t\pm\theta)$ flows through this resistance t average power loss over a whole cycle is

$$\text{Average power loss}=r\frac{1}{T}\int_0^T I^2_{\text{max.}}\sin^2(\omega t\pm\theta)dt \text{ watts} \qquad ($$

The value of the direct-current which produces the same avera power loss as the alternating-current is called the r.m.s. value the alternating-current; that is, the r.m.s. value of a sinusoid alternating-current may be expressed by the relation

$$I_{\text{r.m.s.}}=\sqrt{\frac{1}{T}\int_0^T I^2_{\text{max.}}\sin^2(\omega t\pm\theta)dt}=\frac{I_{\text{max.}}}{\sqrt{2}} \qquad ($$

A similar relation exists between the r.m.s. value of a sinusoid voltage and the maximum value of the voltage, viz.,

$$V_{\text{r.m.s.}}=\frac{V_{\text{max.}}}{\sqrt{2}} \qquad ($$

6. Representation of Sinusoidal Quantities by Diagrams.- Sinusoidal quantities which have the same constant frequenc may be represented by one of the methods illustrated in Fig. The majority of circuit calculations have to do with the relatio between the r.m.s. values of sine-wave quantities and for th reason the method Fig. 2c in which the r.m.s. values of sine-wa quantities are represented by stationary vectors is usually use

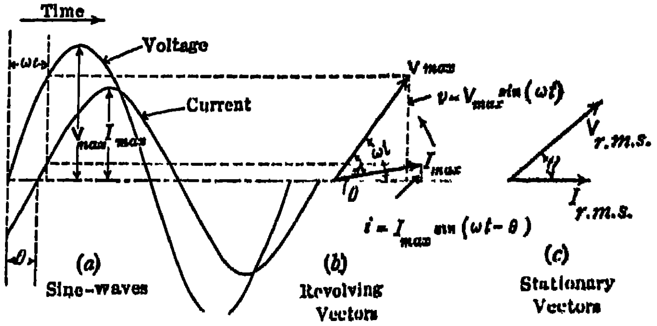


FIG. 2.—Methods for Representing Sine-waves

7. Voltage Drop Across a Circuit Consisting of a Pure Resistance of Constant Value.—If the resistance R of a circuit is abso lutely constant and the inductance and electrostatic capacity of the circuit are zero, a current of the form

$$i=\sqrt{2}I \sin \omega t \tag{10}$$

results in a voltage drop across the circuit of the form

$$v=Ri=R(\sqrt{2}I) \sin (\omega t) \tag{11}$$

where $\sqrt{2}I$ represents the maximum value of the current in the circuit.

In this case the waves representing the voltage and the current are both sine-waves and the two waves are in phase.

The r.m.s. value of the voltage drop across the resistance is

$$V=RI \tag{12}$$

8. Voltage Drop Across a Circuit Consisting of a Pure Inductance of Constant Value.—If the inductance L of a circuit has a constant value throughout a complete cycle of the current and there is no resistance or electrostatic capacity present, a current of the form

$$i=\sqrt{2}I \sin \omega t$$

in the circuit results in a voltage drop across the circuit of the form

$$v=L\frac{di}{dt}=\omega L(\sqrt{2}I) \sin \left(\omega t+\frac{\pi}{2}\right) \tag{13}$$

Putting $\sqrt{2}V=\omega L(\sqrt{2}I)$, we obtain

$$v=\sqrt{2}V \sin \left(\omega t+\frac{\pi}{2}\right) \tag{14}$$

From these relations it follows that a sinusoidal current through a pure inductance produces a sinusoidal voltage drop across the terminals of the inductance and this voltage drop is $\frac{\pi}{2}$ radians or 90 degrees ahead of the current or the current lags the voltage by 90 degrees.

The r.m.s. value of the voltage drop across the inductance is

$$V=\omega LI=X_LI \tag{15}$$

where $X_L=2\pi fL$ is called the inductive reactance of the circuit.

9. Voltage Drop Across the Terminals of a Perfect Condenser of Constant Capacity.—The charge on a condenser of capacity C at any instant of time t may be expressed by the relation

$$q=Cv \tag{16}$$

and the current through it by the relation

$$i=\frac{dq}{dt}=C\frac{dv}{dt} \tag{17}$$

If the current through the condenser is of the form

$$i=\sqrt{2}I\sin(\omega t) \tag{18}$$

the voltage drop across its terminals is

$$v=\frac{1}{C}\int idt=-\frac{\sqrt{2}I}{\omega C}\cos(\omega t)=\frac{\sqrt{2}I}{\omega C}\sin\left(\omega t-\frac{\pi}{2}\right) \tag{19}$$

Putting $\sqrt{2}V=\frac{\sqrt{2}I}{\omega C}$, we obtain

$$v=\sqrt{2}V\sin\left(\omega t-\frac{\pi}{2}\right) \tag{20}$$

It follows that the voltage drop across a perfect condenser of constant capacity is a sinusoidal quantity when the current through it is a sine-wave. The voltage drop lags the current by 90 degrees or the current leads the voltage drop by the same angle.

The r.m.s. value of the voltage drop across the condenser is

$$V=\frac{I}{\omega C}=X_cI \tag{21}$$

where $X_c=\frac{1}{2\pi fC}$ is called the condensive reactance of the circuit.

10. Voltage Drop and Current Relations in a Receiving Circuit Consisting of Resistance, Inductance, and Capacity Connected in Series.—The receiving circuit under consideration is shown in Fig. 3*a*. On account of the series connection the resistance, inductance, and capacity carry the same current. If the current is a sine-wave, the r.m.s. value of the voltage drop RI across the resistance is in phase with the current I; the drop ωLI across the inductance is 90 degrees ahead of the current; and the drop $\frac{I}{\omega C}$ across the condenser lags the current by 90 degrees.

The resultant voltage drop across the circuit is the vector sum of the drops across the resistance, the inductance, and the capacity (see Fig. 3*b*). The resultant drop may be expressed by the relation

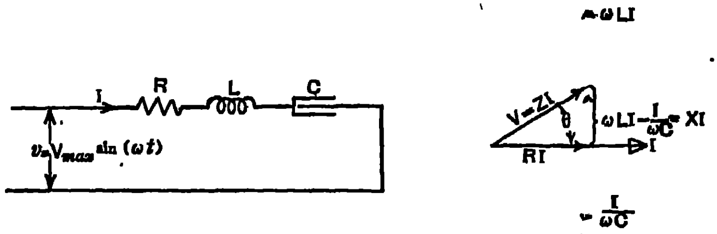
$$V=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}\,I=\sqrt{R^2+X^2}\,I=ZI \tag{22}$$

Where

$$X=\omega L-\frac{1}{\omega C} \tag{23}$$

is the equivalent inductive reactance of the circuit;

$$Z=\sqrt{R^2+X^2} \tag{24}$$



(*a*) (*b*)

FIG. 3.—Ideal Series Circuit and Vector Diagram

is the equivalent impedance of the circuit;

and
$$\theta=\cos^{-1}\frac{R}{Z}=\tan^{-1}\frac{X}{R} \tag{25}$$

is the phase difference between the voltage drop and the current and is called the power factor angle of the circuit.

11. Voltage Drop and Current Relations in a Circuit Consisting of a Pure Resistance, Pure Inductance and Pure Capacity Connected in Parallel.—In this case the voltage drops across the resistance, the inductance, and the capacity are always equal. If the voltage drop across the circuit is a sine-wave, the current through the resistance is in phase with the voltage drop and the currents through the inductance and the capacity are 90 degrees displaced from the voltage drop as shown in Fig. 4*b*.

The resultant current in the line is the vector sum of the currents in the three branches. That is

$$I=\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{1}{\omega L}-\omega C\right)^2}\,V=\sqrt{G^2+B^2}\,V=YV \tag{26}$$

Where

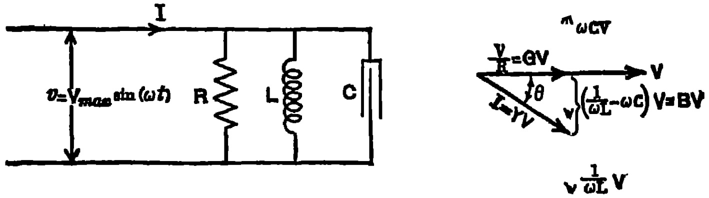
$$G=\frac{1}{R}\text{*} \tag{27}$$

is the equivalent conductance of this particular circuit;

$$B=\frac{1}{\omega L}-\omega C \tag{28}$$

is the equivalent inductive susceptance;

$$Y=\sqrt{G^2+B^2} \tag{29}$$



(a) (b)

FIG. 4.—Ideal Parallel Circuit and Vector Diagram

is the equivalent admittance;

and
$$\theta=\cos^{-1}\frac{G}{Y}=\tan^{-1}\frac{B}{G} \tag{30}$$

is the power factor angle of this circuit.

12. Resonance in Series and Parallel Circuits.—In a series circuit like that shown in Fig. 3*a*, the frequency, inductance, or capacity may be so adjusted that the total reactance of the circuit is zero. Under these conditions

$$\omega L=\frac{1}{\omega C} \tag{31}$$

and for a given impressed voltage the current through the circuit has its maximum r.m.s. value and the voltages across the inductance and the capacity are high, usually much higher than the total impressed voltage. This condition in the circuit is resonance.

---

* It should be noted that the conductance of a circuit carrying an alternating-current is usually not $\frac{1}{R}$. The reader must keep in mind that the circuit in Fig. 4 is an ideal parallel circuit and that equation (36) must be consulted for a perfectly general definition for the conductance of any a-c. circuit.

Resonance may also be obtained in parallel circuits. In a ideal parallel circuit like that in Fig. 4*a*, resonance is obtaine when

$$\frac{1}{\omega L} = \omega C \tag{32}$$

The line current under these conditions has a minimum r.m.s. value and the currents through the inductance and the capacity exceed the line current.

13. General Definitions of Equivalent Impedance, Resistance, Reactance, Admittance, Conductance and Susceptance of a Circuit and the Relations between these Quantities.—

Impedance (Z).—The equivalent impedance of any a-c. circuit is the factor by which the r.m.s. value of the current must be multiplied in order to obtain the r.m.s. value of the resultant voltage drop in the circuit due to its resistance, inductance and capacity.

The term equivalent impedance must not be confused with the leakage impedance of a transformer winding or the synchronous impedance of an a-c. generator. See Articles 48 and 86 in regard to these impedances.

Resistance (R).—The equivalent resistance of any a-c. circuit is the factor by which square of the r.m.s. value of the current must be multiplied in order to obtain the total power in the circuit. The equivalent resistance is also the factor by which the current in the circuit must be multiplied in order to obtain the component of the voltage drop which is in phase with the current.

The equivalent resistance must not be confused with the circuit resistance as obtained by a d-c. resistance bridge.

Inductive reactance (X).—The equivalent inductive reactance of any a-c. circuit is the factor by which the current in the circuit must be multiplied in order to obtain the component of the voltage drop which is 90 degrees ahead of the current. If the inductive reactance of a circuit is negative, the circuit is condensive and the component of the voltage drop due to its reactance lags the current by 90 degrees.

The term equivalent reactance must not be confused with the leakage reactance of a transformer or the synchronous reactance of a generator.

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Admittance (Y).—The equivalent admittance of any a-c. circuit is the factor by which the voltage impressed on the circuit must be multiplied in order to obtain the current in the circuit.

Conductance (G).—The equivalent conductance of any a-c. circuit is the factor by which the voltage impressed on the circuit must be multiplied in order to obtain the component of the current which is in phase with the voltage.

Inductive susceptance (B).—The equivalent inductive susceptance of any a-c. circuit is the factor by which the voltage impressed on the circuit must be multiplied in order to obtain the component of the current which lags the voltage by 90 degrees. If the inductive susceptance of a circuit is negative, the circuit is condensive and the component of the current due to the susceptance leads the voltage by 90 degrees.

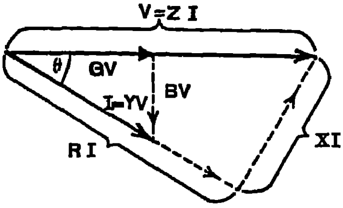


Fig. 5.—Voltage Drop—Current Vector Diagram Showing the Components of the Voltage Drop and the Current

Based on these definitions, the voltage and current components for any inductive circuit may be represented by the vector diagram Fig. 5. (A similar diagram may be drawn for a condensive circuit.) The following perfectly general relations are obtained from the diagram:

$$Y=\frac{1}{Z} \tag{33}$$

$$Z=\sqrt{R^2+X^2} \quad \text{and} \quad Y=\sqrt{G^2+B^2} \tag{34}$$

$$\frac{R}{Z}=\frac{G}{Y}=\cos\theta \tag{35}$$

$$R=\frac{G}{G^2+B^2} \quad \text{and} \quad G=\frac{R}{R^2+X^2} \tag{36}$$

$$\frac{X}{Z}=\frac{B}{Y}=\sin\theta \tag{37}$$

$$X=\frac{B}{G^2+B^2} \quad \text{and} \quad B=\frac{X}{R^2+X^2} \tag{38}$$

From these relations it may be shown that

(1) The inductive reactance of a pure inductance is

$$(+2\pi fL)$$

(2) The inductive reactance of a pure capacity is

$$\left(-\frac{1}{2\pi fC}\right)$$

(3) The inductive susceptance of a pure inductance is

$$\left(+\frac{1}{2\pi fL}\right)$$

(4) The inductive susceptance of a pure capacity is

$$(-2\pi fC)$$

(5) The conductance of a pure resistance is

$$\left(\frac{1}{R}\right)$$

When L is expressed in henrys, C in farads, and R in ohms; then Z, R, and X are in ohms, and Y, G, and B are in mhos.

Equations (33) to (38) enable one to calculate the voltage and current relations in ordinary series and parallel circuits. In series circuits the best procedure is to calculate the equivalent impedance and from it establish the voltage and current relations. In a parallel circuit it is usually best to calculate the admittance of each branch and then proceed to determine the currents in the branches of the circuit.

14. The Average Power in a Single-phase Circuit.—The relations deduced in this article are for sinusoidal voltages and currents. (For the more general relations which hold when the current and the voltage are non-sinusoidal quantities see Article 24 of this chapter.) Let the voltage across a circuit be

$$v=\sqrt{2}V\sin(\omega t) \tag{39}$$

and the current in this circuit

$$i=\sqrt{2}I\sin(\omega t\pm\theta) \tag{40}$$

The instantaneous power in the circuit is then

$$p=vi=\sqrt{2}V\sqrt{2}I\sin\omega t\sin(\omega t\pm\theta) \tag{41}$$

and the average power over a whole cycle is obtained by integrating this expression over a whole cycle and dividing by the time T required for a cycle. This integration gives

$$P=\frac{1}{T}\int_0^T pdt=\frac{1}{T}\int_0^T 2VI \sin (\omega t) \sin (\omega t\pm\theta)dt=VI \cos \theta \quad (42)$$

The average power in a single-phase circuit is therefore equal to the product of the r.m.s. value of the voltage by the r.m.s. value of the current by the cosine of the angle θ. The factor $\cos \theta$ is called the power factor of the circuit. In general, the power factor of a circuit is defined as the ratio of the average power supplied to it (or by it) to the product of the current through it and the voltage across its terminals.

15. Calculation of the Voltage Drop, Current, Power and Power Factor in Combined Series and Parallel Circuits.—To illustrate the method of calculations the combination of series and parallel circuits shown in Fig. 6 will be considered. The constants of the circuit and the impressed voltage and frequency are assumed as known, and are indicated by the proper symbols on the figure.

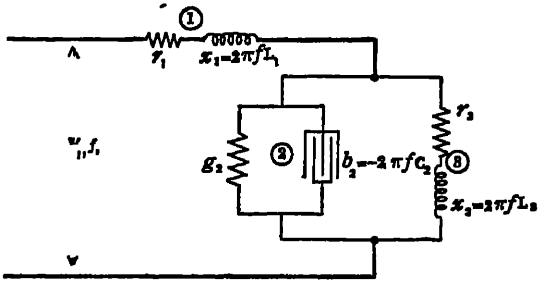


FIG. 6.—Typical Series Parallel Circuit

The problem is to obtain the current, power, and power factor of the whole circuit. The procedure is to

(*a*) Replace the series circuit 3 by a parallel circuit which at the given frequency is equivalent to the series circuit.

(*b*) Replace the circuit 2 and the equivalent parallel circuit of 3 by an equivalent parallel circuit.

(*c*) Replace the equivalent parallel circuit found in *b* by an equivalent series circuit.

(*d*) Replace circuit 1 and that found in *c* by an equivalent series circuit, and then solve for the current, power, and power factor.

These replacements are made as follows:

(a) Constants of the equivalent parallel circuit of circuit 3.—

$$g_3=\frac{r_3}{r_3^2+x_3^2}$$

$$b_3=\frac{x_3}{r_3^2+x_3^2}$$

(b) Constants of equivalent parallel circuit of 2 and 3 combined.—

$$g_{2+3}=g_2+g_3$$

$$b_{2+3}=b_2+b_3$$

(c) Constants of equivalent series circuit of 2 and 3 combined.—

$$r_{2+3}=\frac{g_{2+3}}{g^2_{2+3}+b^2_{2+3}}$$

$$x_{2+3}=\frac{b_{2+3}}{g^2_{2+3}+b^2_{2+3}}$$

(d) Constants of equivalent series circuit 1, 2 and 3 combined.—

$$R=r_1+r_{2+3}$$

$$X=x_1+x_{2+3}$$

Current.—

$$I=\frac{V}{\sqrt{R^2+X^2}}$$

Power factor.—

$$\cos\theta=\frac{R}{\sqrt{R^2+X^2}}$$

Power.—

$$P=VI\cos\theta$$

16. Symbolic Notation.—Circuit calculations are usually made by the symbolic method. Briefly stated the method consists in resolving the vector quantities into components along two axes. The symbolic expression for a vector quantity of the form

$$V=100+j75 \qquad (43)$$

means that the vector V has a component of 100 units length in a positive direction along a given axis and a component of 75 units length in a positive direction along an axis perpendicular to the first axis. The letter j in (43) is used to designate the component along the latter axis.

The numerical value of vector in (43) is

$$V=\sqrt{100^2+75^2} \tag{44}$$

and its phase angle with respect to the first axis is

$$\phi=\tan^{-1}\tfrac{75}{100} \tag{45}$$

***j* Operator.**—The symbol j is used to indicate the real operation of rotating a given vector in a positive direction (counterclockwise) through an angle of 90 degrees. Thus the quantity jV represents a vector which is located 90 degrees ahead of the vector V and which has the same value as V. The quantity jjV represents a vector 90 degrees ahead of the vector jV and is consequently 180 degrees ahead of the vector V. It follows that two operations of j on V results in a new vector equal to V but reversed in direction. If successive operations are represented by powers of j it follows that

$$\begin{aligned} jjV&=j^2V=-V\\ jjjV&=j^3V=-jV\\ jjjjV&=j^4V=V \end{aligned}$$

and that in accordance with this notation $j=\sqrt{-1}$. This does not mean that j is an imaginary quantity for in reality in most cases the symbol j represents an entirely real operation.

On this basis a vector of the form

$$\dot{V}=V\cos\theta+jV\sin\theta$$

must be considered as made up of two components. One component consists of $V\cos\theta$ unit vectors and the other $V\sin\theta$ unit vectors turned through an angle of 90 degrees. The angle θ is the angle between the actual vector and the unit vector.

(Cos $\phi+j$ Sin ϕ) Operator.—When the symbolic expression of a vector of the form

$$\dot{V}=V\cos\theta+jV\sin\theta=V\epsilon^{j\theta}$$

is multiplied by the quantity

$$\cos \phi + j \sin \phi = \epsilon^{j\phi} \text{ (see Demoivrés theorem)} \tag{46}$$

the symbolic expression

$$\dot{V}' = V \cos (\theta + \phi) + jV \sin (\theta + \phi) = V\epsilon^{j(\theta+\phi)}$$

for a new vector is obtained.

The new vector and the original vector have the same lengths for it is readily shown that the square roots of the sums of the squares of their respective components are equal. However, the new vector is located ϕ degrees ahead of the original vector. For these reasons the quantity $\cos \phi + j \sin \phi$ may be considered an operator which turns a vector in a positive direction of ϕ degrees, but does not change the length of the vector.

Impedance operator.—In any circuit the product of the equivalent resistance and the current gives the component of the voltage drop which is in phase with the current and the product of the equivalent inductive reactance and the current gives the component of the voltage drop which is 90 degrees ahead of the current. If the current is selected as the reference axis the symbolic expression for the voltage drop referred to the current is

$$\dot{V} = RI + jXI = (R + jX)I \tag{47}$$

The quantity $R + jX$ is called the impedance operator or simply the symbolic expression for the impedance. The impedance may be represented by any one of the following forms

$$\dot{Z} = R + jX = Z\epsilon^{j\beta} = Z \angle \beta \tag{48}$$

Where $Z = \sqrt{R^2 + X^2}$ is the numerical value of the impedance and

$$\beta = \tan^{-1}\frac{X}{R}.$$

Admittance operator.—In any circuit the product of the equivalent conductance of the circuit and the voltage gives the component of the current which is in phase with the voltage. Similarly the product of the equivalent inductive susceptance of the circuit and the voltage gives the component of the current which lags the voltage by 90 degrees. If the voltage is considered as the reference vector the symbolic expression for the current referred to the voltage is

$$I = GV - jBV = (G - jB)V \tag{49}$$

The quantity $G - jB$ is called the admittance operator or simply the symbolic expression for the admittance. The admittance may be represented by any of the following forms

$$\underset{\cdot}{Y} = G - jB = Y\epsilon^{-j\beta} = Y \angle -\beta \tag{50}$$

Where $Y = \sqrt{G^2 + B^2}$ is the numerical value of the admittance and

$$\beta = \tan^{-1}\frac{B}{G}.$$

Relation between the symbolic expressions for the equivalent impedance and the equivalent admittance of a circuit.—By definition, the equivalent admittance of any circuit is the reciprocal of its equivalent impedance. If a circuit has an equivalent resistance R and an equivalent inductive reactance X the symbolic expression for the admittance of this circuit is

$$\underset{\cdot}{Y} = \frac{1}{\underset{\cdot}{Z}} = \left[\frac{1}{R + jX}\right]\frac{R - jX}{R - jX} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} \tag{51}$$

If the circuit consists of a number of parts in series, then

$$R = r_1 + r_2 + r_3 + \text{etc.}$$

and

$$X = x_1 + x_2 + x_3 + \text{etc.}$$

where r_1, r_2, r_3, etc., are the equivalent resistances of the various parts of the circuit and x_1, x_2, x_3, etc., the inductive reactances of the corresponding parts.

Similarly if the equivalent conductance of a parallel circuit is G, and its equivalent inductive susceptance is B, then

$$\underset{\cdot}{Z} = \frac{1}{\underset{\cdot}{Y}} = \frac{1}{G - jB} = \frac{G}{G^2 + B^2} + j\frac{B}{G^2 + B^2} \tag{52}$$

If the circuit consists of several branches in parallel, then

$$G = g_1 + g_2 + g_3 \text{ etc.}$$

and

$$B = b_1 + b_2 + b_3 \text{ etc.}$$

where g_1, g_2, g_3, etc., are the equivalent conductances of the different branches of the circuit and b_1, b_2, b_3, etc., are the equivalent inductive susceptances of the branches.

17. Solution of the Problem in Article 15 by the Symbolic Method.—

(*a*) **Equivalent admittance of series circuit 3.**—

$$\underset{\cdot}{Y}_3=\frac{r_3}{r_3^2+x_3^2}-j\frac{x_3}{r_3^2+x_3^2}$$

(*b*) **Equivalent admittance of parallel circuits 2 and 3.**—

$$\underset{\cdot}{Y}_{2+3}=(g_2+g_3)-j(b_2+b_3)=g_{2+3}-jb_{2+3}$$

(*c*) **Equivalent impedance of the parallel circuits 2 and 3.**—

$$\underset{\cdot}{Z}_{2+3}=\frac{1}{\underset{\cdot}{Y}_{2+3}}=\frac{g_{2+3}}{g^2_{2+3}+b^2_{2+3}}+j\frac{b_{2+3}}{g^2_{2+3}+b^2_{2+3}}=r_{2+3}+jx_{2+3}$$

(*d*) **Equivalent impedance of the whole circuit.**—

$$\underset{\cdot}{Z}=(r_1+r_{2+3})+j(x_1+x_{2+3})=R+jX$$

Impressed voltage.—The impressed voltage on the circuit is assumed the reference vector, that is

$$\underset{\cdot}{V}=V+j0$$

Current.—

$$\underset{\cdot}{I}=\frac{V}{R+jX}=I_x-jI_y$$

$$I=\sqrt{I_x^2+I_y^2}\quad\text{(numerical value)}$$

Power factor.—

$$\cos\theta=\cos\left(\tan^{-1}\frac{I_y}{I_x}\right)$$

Power.—The power is calculated by the usual method.

$$P=VI\cos\theta$$

The symbolic method for calculating the power by the components of the voltage and current is very confusing and should not be used.

18. Polyphase Circuits.—Large quantities of a-c. power are always transmitted by polyphase circuits. Polyphase generators and motors are better and cheaper than single-phase machines of the same power and a polyphase transmission line (if the number of phases is not too many) may be constructed with less material than a single-phase line transmitting the same power at the same line voltage. Generators and motors may be constructed for any number of phases but usually nothing is gained by making the number of phases greater than three and the same is true of polyphase transmission lines. For these reasons the generation, transmission and utilization of large quantities of a-c. power is usually accomplished by a three-phase system.

19. Voltage Drop, Current, and Power Relations in a Balanced Y-connected Three-phase Load.—A three-phase load is said to be balanced when balanced three-phase voltages (voltages having identical wave shapes and different in phase by 120 degrees) produce balanced line currents.

A balanced Y-connected load is shown in Fig. 7*a*, and the relations between the r.m.s. voltages and r.m.s. currents are shown by the vector diagram Fig. 7*b*. (The double subscript method of indicating the positive sense of the voltages and currents is used.)

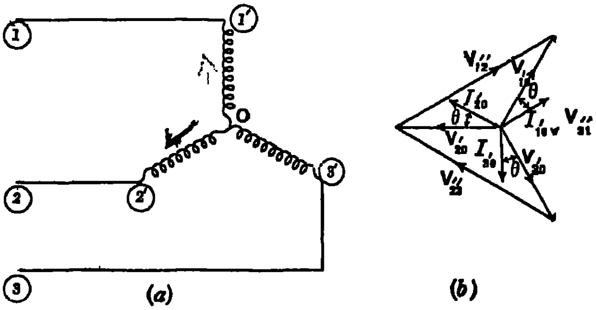


FIG. 7.—Y-connected Load and Vector Diagram

Line voltages and Y-phase voltages.—The line voltages may be expressed in terms of the Y-phase voltages by the relations

$$V_{1'2'} = V_{1'0} + V_{02'}$$

$$V_{2'3'} = V_{2'0} + V_{03'}$$

and

$$V_{3'1'} = V_{3'0} + V_{01'}$$

It may be shown by the vector diagram that the r.m.s. value of the voltage between any two line wires is $\sqrt{3}$ times the r.m.s. value of the Y-phase voltage. That is

$$V_{\text{line}} = \sqrt{3}\,V_{\text{phase}} \qquad (53)$$

Line currents and Y-phase currents.—From Fig. 7*a* it is evident that the r.m.s. value of the current in any line is equal to the r.m.s. value of the Y-phase current. That is

$$I_{\text{line}} = I_{\text{phase}} \tag{54}$$

Power.—A balanced Y-connected three-phase load may be considered equivalent to three single-phase loads across each of which the voltage is equal to line voltage of the three-phase source divided by the $\sqrt{3}$. The total power delivered to a balanced Y-connected load is therefore 3 times the power in one Y-phase. That is

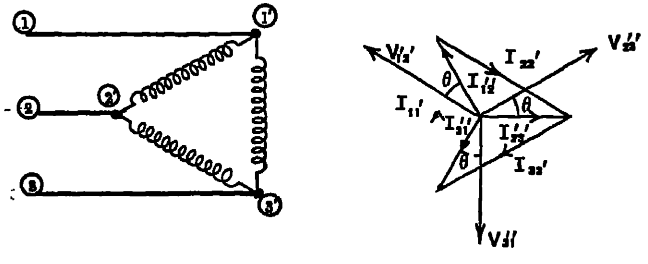
$$P = 3V_{\text{phase}}I_{\text{phase}} \cos \theta_{\text{phase}} \tag{55}$$

or

$$P = \sqrt{3}V_{\text{line}}I_{\text{line}} \cos \theta_{\text{phase}} \tag{56}$$

Note in this expression that θ_{phase} is the phase angle between the Y-phase voltage and Y-phase current and not the angle between the line voltage and the line current.

Voltage drop current and power relations in a balanced Δ-connected three-phase load.—The circuit and the corresponding vector diagram are shown by Figs. 8*a* and 8*b*.



(*a*) (*b*)

FIG. 8.—Δ-connected Load and Vector Diagram

Line voltages and Δ-phase voltages.—From Fig. 8*a*, it is apparent that the line voltages are equal to the Δ-phase voltages. That is

$$V_{\text{line}} = V_{\text{phase}} \tag{57}$$

Line currents and Δ-phase currents.—The line currents and the Δ-phase currents may be expressed by the following relations:

$$I_{11'} = I_{1'2'} + I_{1'3'}$$

$$I_{22'} = I_{2'1'} + I_{2'3'}$$

and

$$I_{33'} = I_{3'2'} + I_{3'1'}$$

Using Fig. 8*b* it may be shown that the r.m.s. value of the current in any line wire is equal to $\sqrt{3}$ times the r.m.s. current in a Δ-phase. That is

$$I_{\text{line}} = \sqrt{3}I_{\text{phase}} \tag{58}$$

Power.—The total power is 3 times the Δ-phase power

$$\therefore P = 3V_{\text{phase}}I_{\text{phase}} \cos \theta_{\text{phase}} \tag{59}$$

or

$$P = \sqrt{3}V_{\text{line}}I_{\text{line}} \cos \theta_{\text{phase}} \tag{60}$$

In this case θ_{phase} is the phase angle between the Δ-phase voltage and the corresponding Δ-phase current.

20. Equivalent Y- and Δ-Circuits.—In most problems having to do with polyphase circuits it is impossible to tell whether the circuits are connected in Y or Δ. The actual connections of the circuits are usually not required, for every balanced Δ-connected circuit is equivalent to a certain balanced Y-connected circuit and vice versa. Calculations involving three-phase circuits are usually carried out with the idea that the circuit is Y-connected.

If the equivalent resistance R_Δ and the equivalent reactance X_Δ of a Δ-phase of a balance Δ-connected load are known it is possible to determine the resistance R_Y and the reactance X_Y of a Y-phase of a balanced Y-connected load equivalent to the Δ-connected load.

The Y-phase resistance is obtained in terms of the Δ-phase resistance on the basis that the power loss in the Y-connected load must equal the power loss in the Δ-connected load. That is

$$3R_YI^2_{\text{line}} = 3R_\Delta\left(\frac{I_{\text{line}}}{\sqrt{3}}\right)^2$$

or

$$R_Y = \frac{R_\Delta}{3} \tag{61}$$

The relation between the reactances is determined from the relation

$$\tan \theta_Y = \frac{X_Y}{R_Y} = \frac{X_\Delta}{R_\Delta}$$

$$\therefore \quad X_Y = \frac{X_\Delta}{3} \tag{62}$$

21. Power and Power Factor Measurements.—

Single-phase.—To determine the power and power factor of a single-phase circuit an ammeter, voltmeter, and wattmeter are required. One method for connecting the meters is shown in Fig. 9. If the current in the circuit is large and the voltage relatively small, corrections must be made for the voltage drops across the ammeter and the current coil of the wattmeter. The power factor of the load is

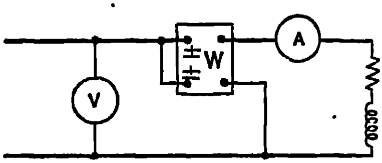


FIG. 9.—Connection of Meters for Measuring the Power in a Single-phase Load

$$\cos\theta = \frac{\text{Wattmeter reading}}{VI}$$

Three-phase.—The average power in a balanced or unbalanced three-phase load may be measured by two wattmeters connected as shown in Fig. 10*a*. The instantaneous power indicated by meter W_1 is

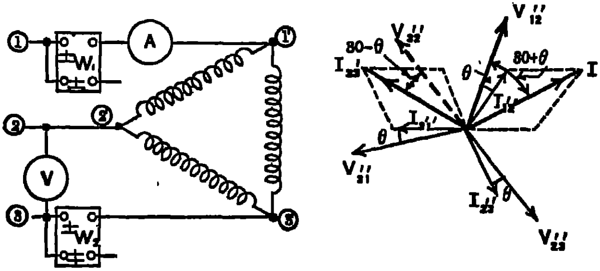
$$p_1 = v_{1'2'}i_{11'}$$

that indicated by meter W_2 is

$$p_2 = v_{3'2'}i_{33'}$$

and the total instantaneous power indicated by the meters is

$$p_1 + p_2 = v_{1'2'}i_{11'} + v_{3'2'}i_{33'} = v_{1'2'}i_{1'2'} + v_{2'3'}i_{2'3'} + v_{3'1'}i_{3'1'}$$



(*a*) (*b*)

FIG. 10.—Connection of Two Wattmeters for Measuring the Power in a Three-phase Load and Vector Diagram

In other words the meters are so connected that the sum of their indications is at all instants equal to the instantaneous power in the three-phase load and the average power indicated by the meters is necessarily equal to average power in the three-phase load. This is true regardless of whether the three-phase load is balanced or unbalanced.

If the load is balanced and the phase rotation of the load voltages, is $V_{1'2'}-V_{2'3'}-V_{3'1'}$ the vector diagram Fig. 10*b* is obtained.

It follows from Figs. 10*a* and 10*b* that the average power indicated by meter W_1, is

$$P_1 = V_{1'2'} I_{11'} \cos (30+\theta_{\text{phase}}) = V_{\text{line}} I_{\text{line}} \cos (30+\theta_{\text{phase}}) \tag{63}$$

and that indicated by meter W_2 is

$$P_2 = V_{3'2'} I_{33'} \cos (30-\theta_{\text{phase}}) = V_{\text{line}} I_{\text{line}} \cos (30-\theta_{\text{phase}}) \tag{64}$$

The sum of the meter indications is

$$P = V_{\text{line}} I_{\text{line}} [\cos (30+\theta_{\text{phase}}) + \cos (30-\theta_{\text{phase}}]$$

$$= \sqrt{3} V_{\text{line}} I_{\text{line}} \cos \theta_{\text{phase}} \tag{65}$$

If θ_{phase} is greater than 60 degrees the power indicated by W_1 is negative and must be subtracted from the indication of W_2 to obtain the total power in the load. The best way to determine whether the meter readings must be added or subtracted is to connect the meters symmetrically as shown in Fig. 10*a*. If under these conditions one of the meter readings is negative, the two readings must be subtracted to obtain the total power.

22. Oscillograph.—The oscillograph is an instrument for studying the actual wave shapes and phase relations of the voltage and the currents in circuits. The essential parts of an oscillograph are shown in Fig. 11. A one-turn low-inertia galvanometer coil carries the current whose wave shape is to be determined. Any motion of the coil, usually called the vibrator, is registered by photographing the path of an intense beam of light reflected from a tiny mirror attached to the vibrator. The vibrator of an ordinary oscillograph may be made to record wave shapes of voltages and currents of frequencies as high as 1000 cycles per second. The natural mechanical period of vibration of a vibrator is approximately 10,000 cycles per second.

Since the current capacity of a vibrator is approximately 0.1 ampere, it is necessary, when a voltage wave is to be determined, to connect in series with the vibrator a resistance (approximately equal to the resistance of a voltmeter required to measure the r.m.s. value of the same voltage). Similarly, to determine the wave shape of a heavy current, a shunt must be used.

To determine the correct phase relations it is necessary to mark the positive and negative terminals of the vibrator. A current entering the positive terminal and leaving the negative terminal is assumed to give a positive throw of the vibrator (for example, a throw to the right in Fig. 11). The polarities of a vibrator are readily determined by using a dry cell in series with a resistance and noting the deflections.

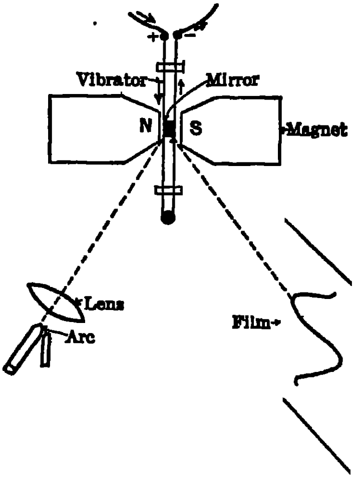


Fig. 11.—Essential Parts of an Oscillograph

23. Fourier's Series.—Any irregular wave varying continuously with time and of constant wave shape and constant frequency may be resolved into a number of sine-waves whose frequencies differ by an integer multiple of the frequency of the irregular wave. If f represents the frequency of an irregular wave the current at any instant of time measured on the irregular wave may be expressed by the series

$$i=\sqrt{2}[I_1 \sin(\omega t+\theta_1)+I_2 \sin(2\omega t+\theta_2)+I_3 \sin(3\omega t+\theta_3)$$
$$+I_4 \sin(4\omega t+\theta_4)+I_5 \sin(5\omega t+\theta_5)+\text{etc.}] \quad (66)$$

Where $\sqrt{2}I_1 \sin(\omega t+\theta_1)$ represents the fundamental or "first harmonic" in the irregular current wave; $\sqrt{2}I_1$ is the maximum value of the fundamental; $\omega=2\pi f$ (the frequency f of the irregular wave and the fundamental are always the same); and θ_1 represents a constant. The second term in the series represents the "second harmonic," the third term the "third harmonic," etc.

24. Power in a Circuit in which the Voltage and the Current are Non-sinusoidal Quantities.—If the voltage drop in a part of a circuit is expressed by the relation

$$v=\sqrt{2}[V_1 \sin(\omega t+\theta'_1)+V_2 \sin(2\omega t+\theta'_2)$$
$$+V_3 \sin(3\omega t+\theta'_3)+\text{etc.}] \quad (67)$$

and the current in it by the relation

$$i=\sqrt{2}[I_1 \sin (\omega t+\theta_1)+I_2 \sin (2\omega t+\theta_2) + I_3 \sin (3\omega t+\theta_3)+ \text{etc.}] \qquad (68)$$

the instantaneous power is the product of the two expressions (vi) and the average value of the power over a complete cycle is

$$P=V_1I_1 \cos (\theta_1-\theta'_1)+V_2I_2 \cos (\theta_2-\theta'_2) + V_3I_3 \cos (\theta_3-\theta'_3)+\text{etc.} \qquad (69)$$

The average power due to each pair of harmonics is independent of the other harmonics present. If any harmonic is absent in either the voltage or the current wave, the average power due to that harmonic is zero.

25. R.M.S. Value of a Non-sinusoidal Wave.—The r.m.s. value of non-sinusoidal wave is obtained by finding the square root of the average square of its instantaneous values over a whole cycle. The r.m.s. values of the voltage and current waves represented by equations (67) and (68) are

$$V=\sqrt{V_1^2+V_2^2+V_3^2+\text{etc.}} \qquad (70)$$

and

$$I=\sqrt{I_1^2+I_2^2+I_3^2+\text{etc.}} \qquad (71)$$

26. Current Produced by a Non-sinusoidal Voltage.—The wave shape of the current produced by an irregular voltage wave depends on the constants of the circuit. If a circuit consists of a pure resistance of constant value an irregular voltage wave produces a current wave of identically the same shape. This is not true when the circuit contains a pure inductance or a pure capacity. The reactance of an inductive circuit having a constant inductance depends on the frequency and for this reason the higher harmonics in the current wave are suppressed with the result that an irregular voltage wave produces a less irregular current wave. In a condensive circuit the reactance becomes less as the frequency is increased and an irregular voltage wave produces a more irregular current wave.

27. Fisher-Hinnen Method of Wave Analysis.—Fig. 12 represents one cycle of an irregular wave in which the harmonics are to be determined.

Analysis for the *n*th harmonic.—In order to determine the *n*th harmonic the following steps must be taken:

(1) Divide one wave length of the irregular wave into n equal parts and erect ordinates at the points of division. For the sake of illustration the wave shown in Fig. 12 is divided into three equal parts ($n=3$), the starting point is at O where the ordinate is zero; the next point of division is at A where the ordinate is positive and equal to AP, and the last point at B where the ordinate is negative and equal to BQ.

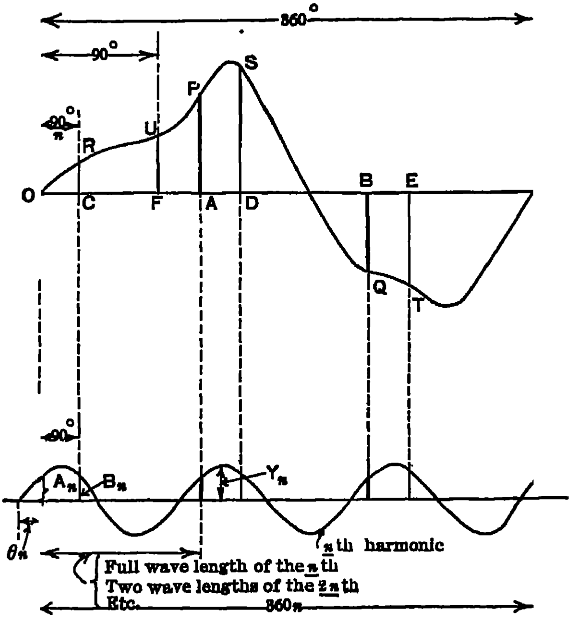


FIG. 12.—Analysis of an Irregular Wave

(2) Determine the algebraic sum of the ordinates erected at these points of division. In the illustration the algebraic sum is $(O+AP-BQ)$.

The algebraic sum of these ordinates may be expressed by the relation

$$S=n(A_n+A_{2n}+A_{3n}+A_{4n}+\text{etc. algebraic sum}) \qquad (72)$$

Where A_n represents the ordinate at any of the points, O, A, or B due to the nth harmonic, A_{2n} is the ordinate at any of these points due to the $2n$th harmonic, etc.

Any harmonic which is not an integer multiple of the nth harmonic contributes nothing to the algebraic sum of the ordinates at the points of division. Thus, if an analysis is made for the third harmonic, the algebraic sum of the ordinates due to the harmonics which are not an integer multiple of the 3rd is

$$\begin{aligned}
&Y_1[\sin\theta_1+\sin(\theta_1+120)+\sin(\theta_1+240)]+\\
&Y_2[\sin\theta_2+\sin(\theta_2+240)+\sin(\theta_2+480)]+\\
&Y_4[\sin\theta_4+\sin(\theta_4+480)+\sin(\theta_4+960)]+\\
&Y_5[\sin\theta_5+\sin(\theta_5+600)+\sin(\theta_5+1200)]+\\
&Y_7[\sin\theta_7+\sin(\theta_7+840)+\sin(\theta_7+1680)]+\\
&Y_8[\sin\theta_8+\sin(\theta_8+960)+\sin(\theta_8+1920)]+\text{etc.}=0 \qquad (73)
\end{aligned}$$

where Y_1, Y_2, Y_4, etc. are the maximum values of the harmonics.

(3) Erect another set of ordinates spaced 90 degrees of the nth harmonic $\left(\frac{90}{n}\text{ degrees of the fundamental}\right)$ ahead of the ordinates obtained in 1. In the illustration these ordinates are CR, DS, and ET.

(4) Determine the algebraic sum of the ordinates obtained in 3. In the illustration this sum is $S'=CR+DS-ET$. The algebraic sum of the ordinates may be expressed by the relation

$$S'=n(B_n+B_{2n}+B_{3n}+\text{etc. algebraic sum}) \qquad (74)$$

where B_n represents the ordinate of the nth harmonic at any of the points of division, C, D, or E, and B_{2n} represents the ordinate at any of these points due to the $2n$th harmonic, etc.

Here again the harmonics which are not an integer multiple of the nth harmonic contribute nothing to the algebraic sum of the ordinates.

It is evident from equations (72) and (74) that in order to determine the ordinates of the nth harmonic at the points of division it is necessary to determine, first, the ordinates of the $2n$th, $3n$th, $4n$th, etc., harmonics. For this reason an irregular wave is always analyzed first for the highest harmonic which is likely to be present.

(5) If the algebraic sums of $A_{2n}+A_{3n}+A_{4n}+$etc., and $B_{2n}+B_{3n}+B_{4n}+$etc., are known, it is easy to determine by equations (72) and (74) the ordinates A_n and B_n of the nth harmonic.

From Fig. 12 it is evident that these ordinates are

$$A_n = Y_n \sin \theta_n \tag{75}$$

and

$$B_n = Y_n \sin (\theta_n + 90) = Y_n \cos \theta_n \tag{76}$$

From these relations it follows that the maximum value of the nth harmonic is

$$Y_n = \sqrt{A_n^2 + B_n^2} \tag{77}$$

and that

$$\theta_n = \tan^{-1} \frac{A_n}{B_n} \tag{78}$$

where θ_n is considered positive when it is measured to the left of the origin O.

Determination of the fundamental.—After the higher harmonics have been determined it is comparatively easy to find the maximum value and the phase relation of the fundamental.

At the point O of the irregular wave the algebraic sum of the ordinates of all the harmonics is zero. That is

$$A_1 + A_2 + A_3 + \text{etc.} = 0 \tag{79}$$

where A_1 represents the ordinate of the fundamental at this point.

At the point F, located 90 degrees ahead of O, the ordinate is equal to the algebraic sum of the ordinates of all the harmonics at this point. That is

$$C_1 + C_2 + C_3 + \text{etc.} = FU \tag{80}$$

where C_1 represents the ordinate of the fundamental at this point and the ordinates C_2, C_3, etc., must be calculated from the known harmonics.

Now
$$A_1 = Y_1 \sin \theta_1 \tag{81}$$

and
$$C_1 = Y_1 \sin (\theta_1 + 90) = Y_1 \cos \theta_1 \tag{82}$$

Therefore
$$Y_1 = \sqrt{A_1^2 + C_1^2} \tag{83}$$

and
$$\theta_1 = \tan^{-1} \frac{A_1}{C_1} \tag{84}$$

PROBLEMS

✓ **1.** A current of the form $i = 100 \sin (377t + 1.5)$ flows in a given circuit [the angle $(377t + 1.5)$ is expressed in radians when t is expressed in seconds]. (*a*) Plot one complete cycle of the wave. (*b*) Determine: (1) the frequency of the current; (2) the period of one cycle; (3) the maximum value of the current; (5) the average value of the current for a half cycle; (6) the crest factor; (7) the form factor; and (8) the phase of the first ascending zero value of the current referred to the origin of time. (*c*) Determine the equation for the instantaneous current in which the angle is expressed in degrees instead of radians.

2. Thirty-six equally spaced ordinates (10 degrees apart) of a 25-cycle current wave have the following values:

i_{0}	0 00	i_{130}	0.7	i_{260}	− 7.18
i_{10}	+ 3 84	i_{140}	− 2.16	i_{270}	− 7.33
i_{20}	+ 6 93	i_{150}	− 3 48	i_{280}	− 5.78
i_{30}	+10.75	i_{160}	− 2 50	i_{290}	− 3.91
i_{40}	+15 72	i_{170}	− 0 77	i_{300}	− 3.87
i_{50}	+20 07	i_{180}	− 0.68	i_{310}	− 6.52
i_{60}	+21 33	i_{190}	− 3 38	i_{320}	−10.1
i_{70}	+18.42	i_{200}	− 7 39	i_{330}	−11.74
i_{80}	+13.0	i_{210}	−10 07	i_{340}	− 9.76
i_{90}	+ 8.01	i_{220}	− 9 90	i_{350}	− 5.05
i_{100}	+ 5 32	i_{230}	− 7.81	i_{360}	0.00
i_{110}	+ 4.37	i_{240}	− 6 11		
i_{120}	+ 3.19	i_{250}	− 6 16		

Plot the waves on a large sheet of cross-section paper (see Problem 24) and determine: (*a*) the r.m.s. value of the current; (*b*) the average value of the current; (*c*) the crest factor; and (*d*) the form factor.

3. A sinusoidal electromotive force of the form $e = 120 \sin (377t)$ acts around a closed circuit of which the resistance is 3 ohms and of which the reactance is zero. (*a*) Write an equation for the instantaneous current in the circuit. (*b*) Determine: (1) the phase difference between the e.m.f. and the current; and (2) the frequency of the e.m.f.

4. An e.m.f. of the form $e = 120 \sin 2\pi ft$ acts in a coil which has its terminals connected together. The electric circuit of the coil has a self-inductance of 0.2 henry and has no appreciable resistance or capacity. The power loss in the magnetic circuit of the coil is negligible. (*a*) Write an equation for the instantaneous current in the coil. (*b*) Determine: (1) the phase difference between the e.m.f. and the current; and (2) the power factor of the coil.

5. An e.m.f. of the form $e = 120 \sin 2\pi ft$ acts around a circuit consisting of a perfect condenser having a capacity of 42 microfarads. The resistance and the inductance of the circuit are negligible. (*a*) Write an equation for the instantaneous current in the circuit. (*b*) Determine: (1) the phase difference between the e.m.f. and the current; and (2) the power factor of the condenser.

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6. A current of the form $i = 85 \sin (2\pi ft + \phi)$ flows in a given coil. The coil has an equivalent resistance of 5 ohms and an equivalent inductive reactance of 2 ohms. (*a*) Write the equation for the instantaneous voltage drop across the terminals of the coil. (*b*) Determine the power factor of the coil. (*c*) Construct a complete vector diagram showing the impressed voltage, the *RI* drop, the *XI* drop, and the current in the coil.

7. The instantaneous voltage drop across the terminals of a 42-microfarad condenser is

$$v = 100 \sin \left(2\pi ft - \frac{\pi}{6}\right)$$

The power loss in the condenser is 30 watts. The frequency is 60 cycles. (*a*) Write the equation for the instantaneous current through the condenser. (*b*) Calculate the power factor of the condenser. (*c*) Construct a vector diagram showing the voltage drop and the current.

8. An air-core coil is connected to a 110-volt (r.m.s. value) 60-cycle supply. The line current is 2.80 amperes (r.m.s. value) and the power loss in the coil is 86.2 watts. (*a*) Calculate the equivalent inductance of the coil. (*b*) Calculate the power loss in the coil when the impressed voltage is 110 volts and the frequency 30 cycles. (*c*) What assumption did you make in part (*b*)?

9. A 100-microfarad condenser is connected to a 110-volt 60-cycle supply. The power loss in the condenser is 60 watts. Calculate: (*a*) the charging current; (*b*) the leakage current; and (*c*) the power factor of the condenser.

10. An air-core reactor having a constant effective resistance (equivalent a-c. resistance) of 2 ohms and a constant self-inductance (equivalent inductance) of 0.9 henry is connected in series with a perfect condenser having 62 microfarads capacity to a 100-volt (r.m.s. value) constant voltage supply. (*a*) Plot a curve showing the relation between r.m.s. values of the line current and frequency. (*b*) What is the maximum r.m.s. value of the current and at what frequency and power factor does it occur? (*d*) Draw a complete voltage drop-current vector diagram for the frequency at which the current has the maximum r.m.s. value.

11. The reactor and the condenser in Problem 10 are connected in parallel to a 100-volt constant voltage a-c. supply. (*a*) Plot a curve showing the relation between r.m.s. values of line current and frequency. (*b*) At what frequency and what power factor is the r.m.s. value of the line current a minimum? (*c*) Draw a complete voltage drop-current vector diagram for the frequency at which the line current has the minimum r.m.s. value.

12. The following measurements were made on a single-phase circuit of which the equivalent resistance, inductance and capacity are unknown:

Voltage	Frequency	Current	Power
110	d-c.	0	0
110	60-cycles	8.95 amps.	80 watts
110	69	110.00 (max. current)	12,100
110	80	8.56	73.2

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PROBLEMS

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i_{30}	+10 75	i_{160}	− 2.50	i_{290}	− 3 91
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i_{60}	+21.33	i_{190}	− 3 38	i_{320}	−10 1
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i_{80}	+13.0	i_{210}	−10.07	i_{340}	− 9 76
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i_{110}	+ 4.37	i_{240}	− 6 11		
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Plot the waves on a large sheet of cross-section paper (see Problem 24) and determine: (*a*) the r.m.s. value of the current; (*b*) the average value of the current; (*c*) the crest factor; and (*d*) the form factor.

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5. An e.m.f. of the form $e = 120 \sin 2\pi ft$ acts around a circuit consisting of a perfect condenser having a capacity of 42 microfarads. The resistance and the inductance of the circuit are negligible. (*a*) Write an equation for the instantaneous current in the circuit. (*b*) Determine: (1) the phase difference between the e.m.f. and the current; and (2) the power factor of the condenser.

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6. A current of the form $i = 85 \sin (2\pi ft + \phi)$ flows in a given coil. The coil has an equivalent resistance of 5 ohms and an equivalent inductive reactance of 2 ohms. (*a*) Write the equation for the instantaneous voltage drop across the terminals of the coil. (*b*) Determine the power factor of the coil. (*c*) Construct a complete vector diagram showing the impressed voltage, the RI drop, the XI drop, and the current in the coil.

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10. An air-core reactor having a constant effective resistance (equivalent a-c. resistance) of 2 ohms and a constant self-inductance (equivalent inductance) of 0.9 henry is connected in series with a perfect condenser having 62 microfarads capacity to a 100-volt (r.m.s. value) constant voltage supply. (*a*) Plot a curve showing the relation between r.m.s. values of the line current and frequency. (*b*) What is the maximum r.m.s. value of the current and at what frequency and power factor does it occur? (*d*) Draw a complete voltage drop-current vector diagram for the frequency at which the current has the maximum r.m.s. value.

11. The reactor and the condenser in Problem 10 are connected in parallel to a 100-volt constant voltage a-c. supply. (*a*) Plot a curve showing the relation between r.m.s. values of line current and frequency. (*b*) At what frequency and what power factor is the r.m.s. value of the line current a minimum? (*c*) Draw a complete voltage drop-current vector diagram for the frequency at which the line current has the minimum r.m.s. value.

12. The following measurements were made on a single-phase circuit of which the equivalent resistance, inductance and capacity are unknown:

Voltage	Frequency	Current	Power
110	d-c.	0	0
110	60-cycles	8 95 amps.	80 watts
110	69	110.00 (max. current)	12,100
110	80	8.56	73.2

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(*a*) Determine the nature of the circuit and the constants of the same. (*b*) What assumptions did you make?

13. An impedance of $4 + j7$ ohms is connected in series with an impedance of $10 + j2$ ohms to a 120-volt a-c. generator having negligible impedance. Determine: (*a*) the symbolic expression for the impedance of the whole circuit; (*b*) the symbolic expression for the admittance of the whole circuit; and (*d*) the actual values of the equivalent impedance, equivalent admittance, and the current.

14. An impedance of $6 + j2$ ohms is connected in parallel with an impedance of $2 - j6$ ohms to a 75-volt a-c. generator having negligible impedance. Determine: (*a*) the symbolic expression for the equivalent admittance of this circuit; (*b*) the symbolic expression for the equivalent impedance of the whole circuit; (*c*) the symbolic expressions for the currents in the different parts of the circuit; and (*d*) the numerical values of the currents in the different parts of the circuit.

15. The following results were obtained from a test on a leaky condenser:

Impressed voltage, 220 volts;
Frequency, 58 cycles;
Current input, 2 amps.;
Power loss in the condenser 52 watts.

(*a*) Calculate the values of a pure resistance and a pure capacity which when connected in series are equivalent (at 58 cycles) to the leaky condenser. (*b*) Calculate the values of a pure resistance and a pure capacity which when connected in parallel are equivalent to the leaky condenser.

16. A pure resistance and a perfect coil are connected in parallel across a 220-volt, 60-cycle supply. The current through the resistance is 14.6 amperes and that through the coil 10.2 amperes. The total line current is 18.5 amperes. Calculate the frequency which would result in equal currents in the coil and the resistance.

17. A reactor having an equivalent a-c. resistance of 0.6 ohm and an equivalent inductive reactance of 5.2 ohms is connected in series with an incandescent lamp load having an equivalent resistance of 10.8 ohms and zero reactance to a 110-volt, 60-cycle supply. Determine the capacity of a perfect condenser which must be connected in parallel with this inductive circuit to obtain unity power factor for the combined circuit.

18. Two wattmeters *A* and *B* are connected symmetrically for measuring the power in a balanced three-phase load. The r.m.s. value of the line voltage is 220 volts; the delta-phase power factor is 0.92 lagging; and the total power delivered (all phases) is 40 kilowatts. (*a*) Assume a phase rotation for the line voltages and determine the power indicated by each meter. (*b*) Determine the power indicated by each meter when the phase rotation is opposite that assumed in part *a*.

19. Two similar wattmeters are connected symmetrically for measuring the power in a balanced three-phase load. One of the meters indicates 8250 watts and the other 2730 watts. (*a*) Calculate the power factor of the load. (*b*) Calculate: (1) the phase angle between the delta-phase voltage and the delta-phase current; (2) the phase angle between the Y-phase voltage and the Y-phase current; and (3) the voltage between lines 1 and 2 and the current in line 1. (*c*) How many answers are possible in part (3) or (*b*)?

20. A balanced Y-connected load and a balanced Δ-connected load are connected in parallel to a balanced three-phase supply. The Y-connected load has an equivalent Y-phase resistance of 2.1 ohms and an equivalent Y-phase inductive reactance of 1.6 ohms. The Δ-connected load has an equivalent Δ-phase resistance of 3.4 ohms and an equivalent Δ-phase inductive reactance of 8.1 ohms. Calculate: (*a*) the constants of a Y-connected load which is equivalent to the two loads in parallel; (*b*) the constants of a Δ-connected load which is equivalent to the two loads in parallel.

21. The total power input to a three-phase induction motor is 7540 watts. The motor operates at a line voltage of 440 volts and at a power factor of 80 per cent lagging. Calculate: (*a*) the Y-phase voltage; (*b*) the Δ-phase voltage; (*c*) the Y-phase power; (*d*) the Δ-phase power; (*e*) the Y-phase power factor; (*f*) the Δ-phase power factor; (*g*) the Y-phase current; (*h*) the Δ-phase current; and (*i*) the line current.

22. The voltage applied to a circuit is of the form

$$v=105\sin\left(\omega t+\frac{\pi}{8}\right)+72\sin\left(3\omega t+\frac{\pi}{6}\right)+51\sin\left(5\omega t+\frac{\pi}{2}\right)$$

and the current in the circuit has the form

$$i=37.5\sin\left(\omega t+\frac{\pi}{6}\right)+10.8\sin\left(5\omega t+\frac{\pi}{6}\right)$$

Calculate: (*a*) the r.m.s. value of the voltage; (*b*) the r.m.s. value of the current; and (*c*) the power in the circuit.

23. The voltage induced in one phase of a three-phase alternator has the form

$$e=211\sin(\omega t+\theta_1)+8.7\sin(3\omega t+\theta_3)$$

Calculate: (*a*) the terminal voltage on open circuit when the phases of the alternator are Y-connected; (*b*) the ratio between the r.m.s. value of the line voltage and the r.m.s. value of the Y-phase voltage; (*c*) the terminal voltage on open circuit when the alternator is Δ-connected; and (*d*) the ratio between the r.m.s. value of the line current and the r.m.s. value of the Δ-phase current. Assume in part *d* that the line current has an r.m.s. value of 250 amperes and that the equivalent Δ-phase resistance and the inductive reactance (the reactance due to the fundamental frequency) are equal to 0.01 ohm and 3.8 ohms respectively.

24. (*A*) Analyze the wave specified in Problem 2 for all harmonics of a lower order than the 7th.

(*B*) Write the equation for the instantaneous current.

CHAPTER II

TRANSFORMER CONSTRUCTION

A transformer is a device used for raising or lowering the voltage of an a-c. power supply without the use of mechanically revolving parts. In its simplest form it consists of two separate windings wound on a common laminated iron core (Fig. 13). Electrical power is transferred from the supply circuit to the load circuit through the medium of the magnetic circuit. The transformer winding which is connected to the supply is called the *primary winding*, and the one connected to the load is called the *secondary winding*. Either winding may be used as primary or secondary, provided the windings are operated at the voltage for which they are designed. The windings are sometimes designated as the *high-voltage winding* and the *low-voltage winding*. The ratio of the turns on the high-voltage winding to the turns on the low-voltage winding is called the *turn ratio* (see A. I. E. E. Standardization Rules).

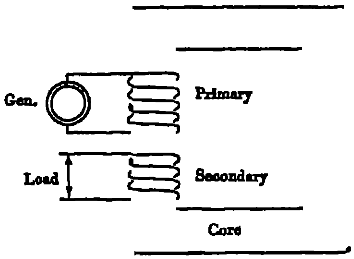


FIG. 13.—Essential Parts of a Transformer

28. Types of Transformers.—For a given weight of active material in a transformer the relative proportions of iron and copper depend on the type of core used. The transformer shown in Fig. 13 is known as a *core-type* transformer and that in Fig. 14 as a *shell-type* transformer. The main difference between the two types is that in the core-type construction the electric circuits link the same iron magnetic circuit while in the shell-type construction the electric circuits are linked by several iron magnetic circuits. The core-type construction permits the use of a relatively small quantity of iron, but requires correspondingly more copper than a shell-type transformer, of equal rating. Both types may be designed for high voltage, high capacity, and high efficiency. The windings on a core-type transformer are easier to insulate than those on a shell-type transformer, but the shell-type transformer is mechanically much stronger. For these reasons high-voltage testing transformers are usually of the core-type and when large quantities of power are to be transformed the shell-type construction is preferred.

29. Windings on Transformers.—

Core-type (Fig. 15).—The high-voltage winding on a core-type transformer consists of a large number of separate coils connected in series and spaced by insulating spacers to permit the circulation of oil between the windings. The low-voltage winding is usually wound on a cylinder of insulating material and occupies the space close to the iron core. The high-voltage winding surrounds the low-voltage winding and is separated from it by a cylinder of insulating material.

Shell-type (Figs. 16, 17, 18, 19).—Both high-voltage and low-voltage windings consist of pan-cake-shaped coils (Fig. 16) connected in series. This form of coil has a large surface exposed to the cooling medium and permits interleaving of the high-voltage and the low-voltage windings (Fig. 17). Fig. 18 illustrates how the coils are assembled and Fig. 19 represents a completely assembled shell-type transformer.

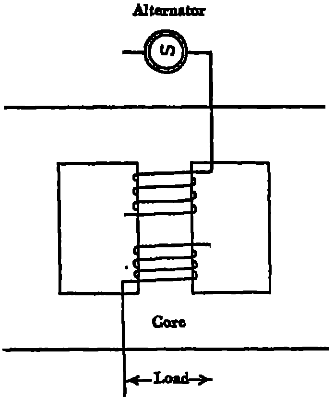
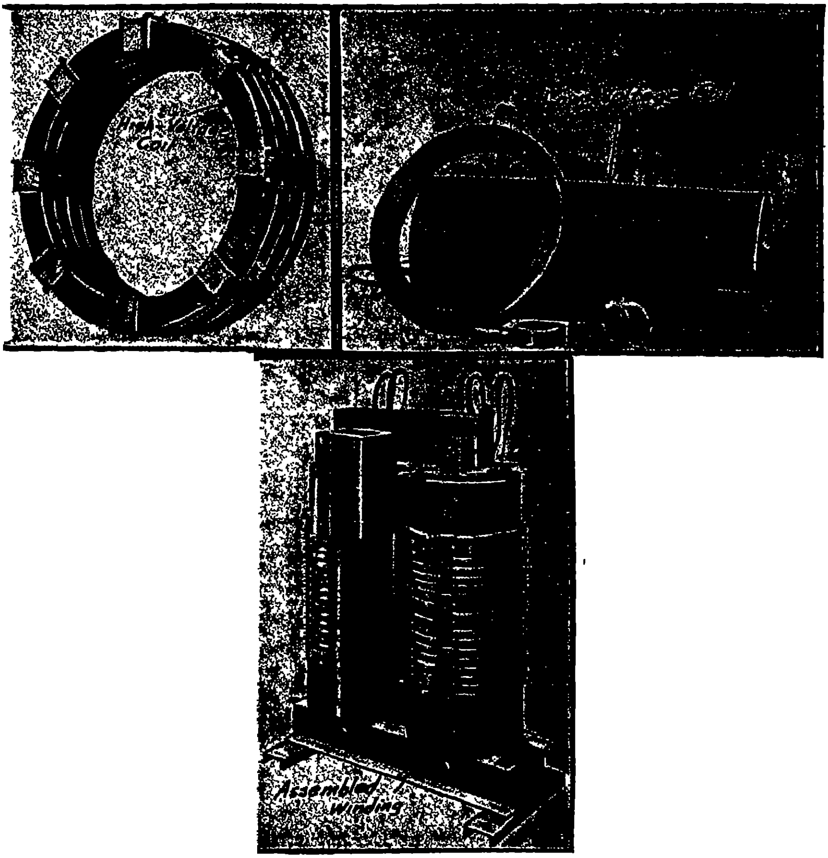


FIG. 14.—Shell-type Transformer

30. Transformer Insulation.—The turns and the coils of a transformer must be thoroughly insulated from each other and from the transformer core. A failure in the insulation usually ruins the transformer and may endanger the life of the operator. Thus in Fig. 20, if the high-voltage circuit is grounded at *a* and the transformer insulation fails at *b*, a high-voltage may exist between *c* and the ground. A person in contact with any part of the low-voltage circuit might under these conditions receive a serious shock.

The proper design of the insulation of all high-voltage machines presents many difficulties, due largely to the fact that the materials available for insulating purposes have uncertain characteristics.

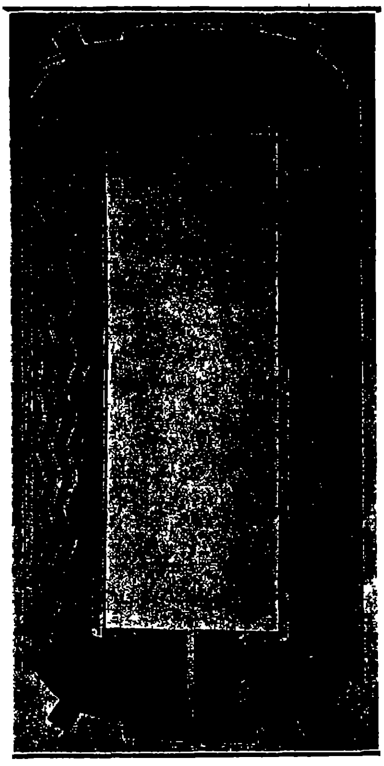


(*By the courtesy of the Westinghouse Co.*)

FIG. 15.—Core-type Transformer

Good insulation must have (1) high dielectric strength, (2) high mechanical strength, (3) high electrical resistance, (4) high heat-resisting qualities, and (5) constant chemical composition. Mica is probably the best insulator known, but it has the undesirable characteristic of being brittle. There are very many insulating materials available and it is beyond the scope of this book to describe them in detail. It might be said that most of them are far from ideal insulators.

The insulation of a transformer winding usually occupies more space than the copper in the winding. In addition to the insulation, space must be allowed between the coils and the insulation for the circulation of the cooling medium. Both air and oil are used for cooling purposes. Oil has an advantage in that it serves the double purpose of cooling and insulating. The coils, insulation, and oil passages of a transformer are shown in Figs. 15 and 18.



(*By the courtesy of the Westinghouse Co.*)

FIG. 16.—Pancake-shaped Transformer Coil and Wavy Spacers

End turn insulation.—On account of the distributed capacity and inductance of a transformer winding, the end turns (the turns nearest the terminals) must be more thoroughly insulated than those in the body of the winding (see Fig. 21*a*). A given transformer winding may be represented by the circuit in Fig. 21*b*. The electrostatic capacity between the successive turns and the core is equivalent to electrostatic condensers connected between each turn and the core.

In the discussion which follows it is assumed that initially both the winding and the core are at ground potential. On this assumption when switch *a* is closed (*b* open) condenser 1′ will be instantly charged to the potential of line *A*. Condenser 2′ will assume line potential later than condenser 1′, since the current required to charge this condenser must flow through the inductance of coil 1. Consequently during a short interval of time the full line potential will exist between turn 1 and turn 2, and the insulation between these two turns must be designed accordingly. The same state of affairs occurs, to a diminishing degree, between the successive turns 2, 3, etc. Similarly, if switch *b* is closed (*a* open), the insulation between turns 5 and 4 will be subjected to the potential of line *B*. Therefore the end turns of both ends of a high-voltage winding must have extra insulation.

High-voltage transformer bushings.—The windings of practically all high-voltage transformers are immersed in oil and placed in moisture-proof tanks. Where the high-voltage leads enter the tank special insulators are required (see Fig. 22*a*). The insulating bushing here represented consists of alternate layers of insulation and tinfoil protected by porcelain insulators on the outside. The successive layers of insulation and tinfoil are equivalent to a large number of condensers connected in series (see Fig. 22*b*). The lengths of the different layers are so proportioned that the condensers formed by them have equal capacities and any potential difference between the transformer terminal and the tank is distributed equally among the condensers with the result that the insulation is stressed uniformly. A terminal of the same dimensions with the tinfoil omitted and homogeneous insulation substituted would probably fail, due to the high-voltage gradient in the vicinity of the conductor. Another type of bushing, not described in detail in this book, consists of an oil-filled porcelain cylinder surrounding a metallic conductor.

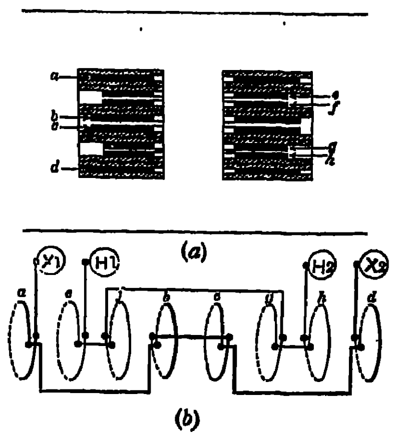
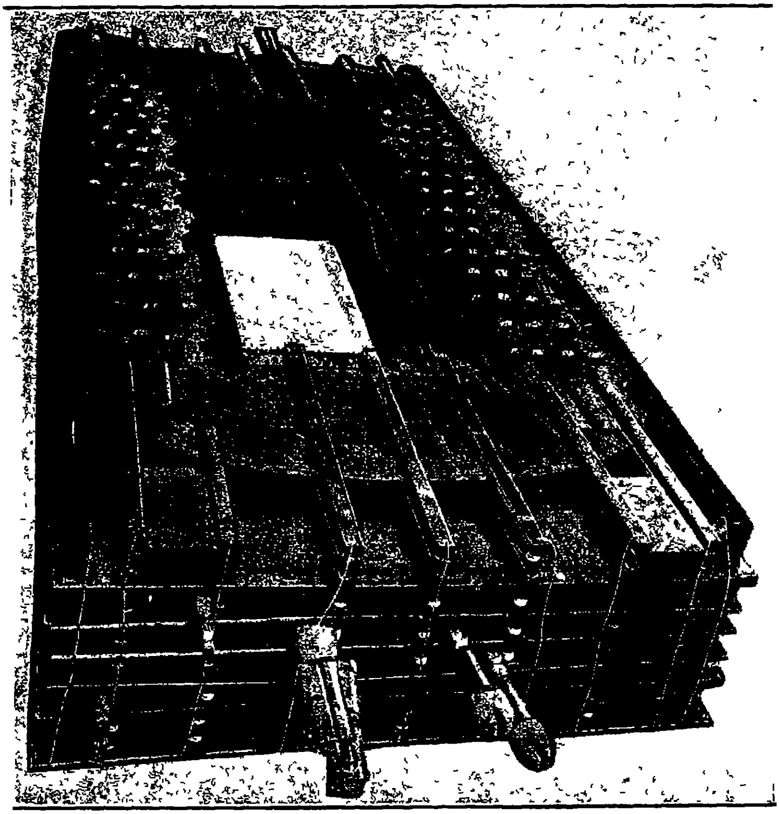


Fig. 17.—Diagrams Showing Locations and Connections of Coils of a Transformer. Coils *a* to *d* belong to the Low-voltage Winding and Coils *e* to *h* to the High-voltage Winding

31. Transformer Cores.—The core in a transformer forms a magnetic connecting link between the primary and secondary windings. Any transfer of power between the two windings must take place through the magnetic field in the core and for this reason the core must be so constructed that the transfer of power is accomplished with a small loss of power. On this basis it would seem that an air-core transformer is ideal because the losses in its magnetic circuit are zero, but another factor must be taken into account. The magnetic flux required in a power transformer is very large and the ampere-turns necessary to produce this flux in air would be excessive. For this reason cores for power transformers are always constructed of iron having a high permeability.



(By the courtesy of the General Electric Co.)

FIG. 18.—Assembled Winding of a Shell-type Transformer

The losses in transformer cores are due to eddy-currents and magnetic hysteresis. The eddy-currents are set up by induced e.m.f.'s acting around closed circuits in the core material. That is, the core material may be considered as made up of an infinite number of closed circuits similar to the secondary winding circuit of the transformer. Naturally the magnitude of the eddy-currents and the losses produced by them depend on the electrical resistance of the material as well as on the construction of the core. A core made of high-resistance material has a much lower eddy-current loss than a similar core constructed of low-resistance material and used under similar conditions. That this is so follows from the fact that the eddy-currents vary as some inverse function of the electrical resistance and consequently the losses produced by these currents which depend on the square of the currents must vary as some inverse function of the square of the electrical resistance. To keep the eddy-current losses small transformer cores are always constructed of laminated iron. The laminations are approximately 0.0141 inch thick and the surfaces of the laminations are varnished so as to form a high electrical resistance between adjacent laminations.

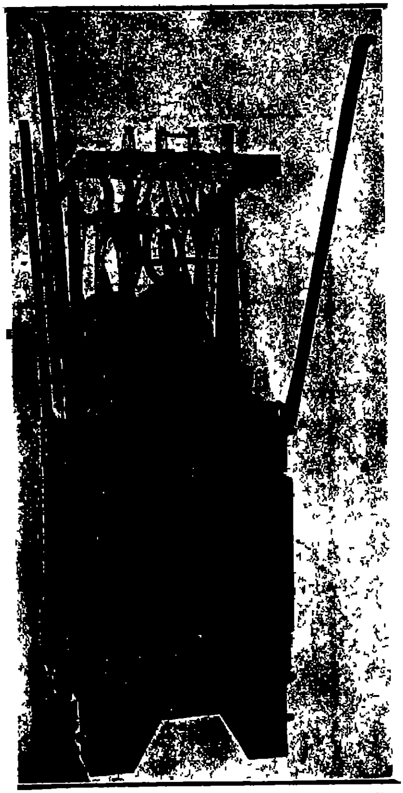

(By the courtesy of the Westinghouse Co.)

Fig. 19.—Shell-type Transformer

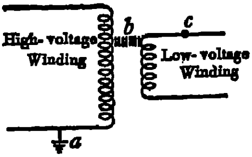


Fig. 20.—Failure of Transformer Insulation

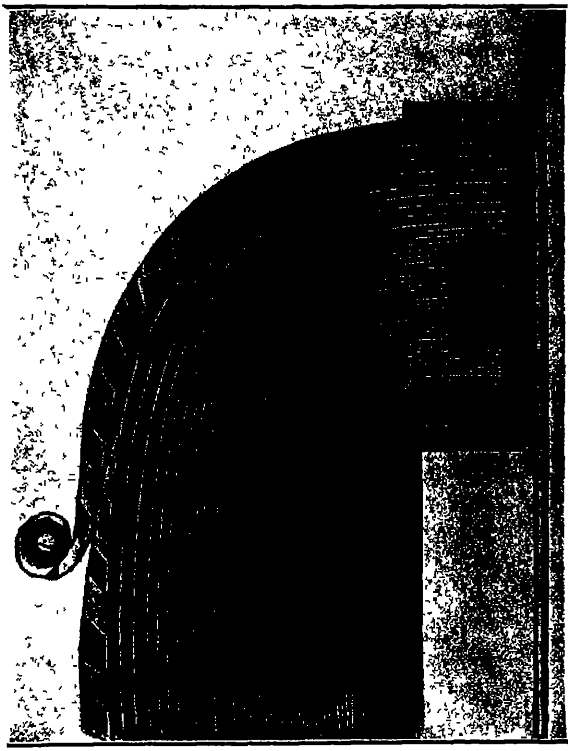
The eddy-current losses may be calculated by the relation

$$P_e = K_e t^2 f^2 B_m{}^2 W$$

where K_e is a factor depending on the kind of material, t is the thickness of the laminations, f is the frequency, B_m is the maximum flux density, and W is the weight of material. (For the proof of this formula and its application to actual cases the reader is referred to Pender's Handbook.)

The magnetic hysteresis loss may be calculated by the relation

$$P_h = K_h f B_m^{1.6} W$$



(*By the courtesy of the Westinghouse Co.*)

Fig. 21*a*.—Part of Transformer Coil Showing End-turn Insulation

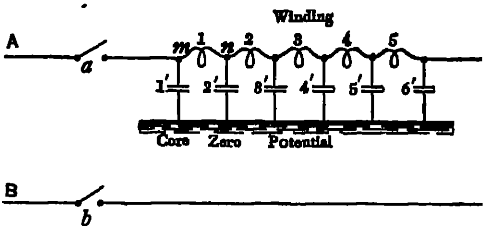
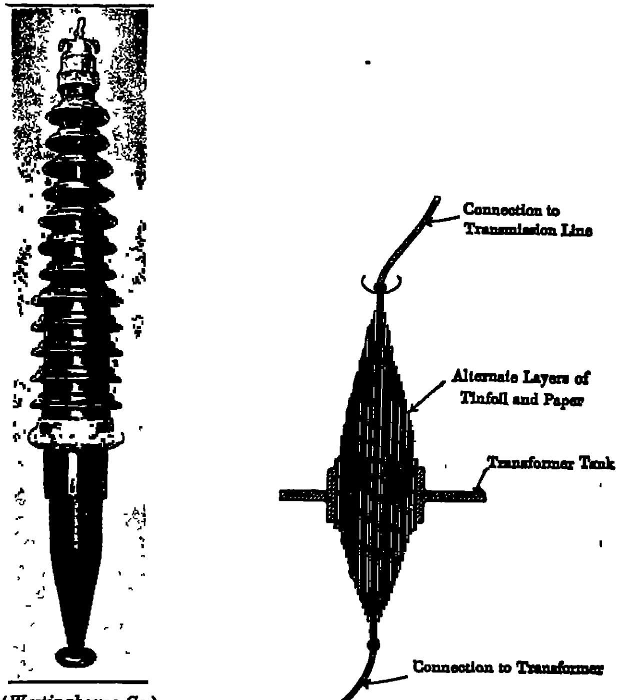


Fig. 21*b*.—Equivalent Circuit of a Transformer Winding

where K_h is a factor depending on the kind of material, f is the frequency, B_m is the maximum flux density, and W is the weight of the material.

The reluctance of the core depends on the kind of material, the length of the magnetic circuit, and on the section area of the magnetic circuit. When the magnetic circuit has a constant section area and is made of uniform material the reluctance may be calculated by the relation


Connection to Transmission Line
Alternate Layers of Tinfoil and Paper
Transformer Tank
Connection to Transformer

(*Westinghouse Co.*)

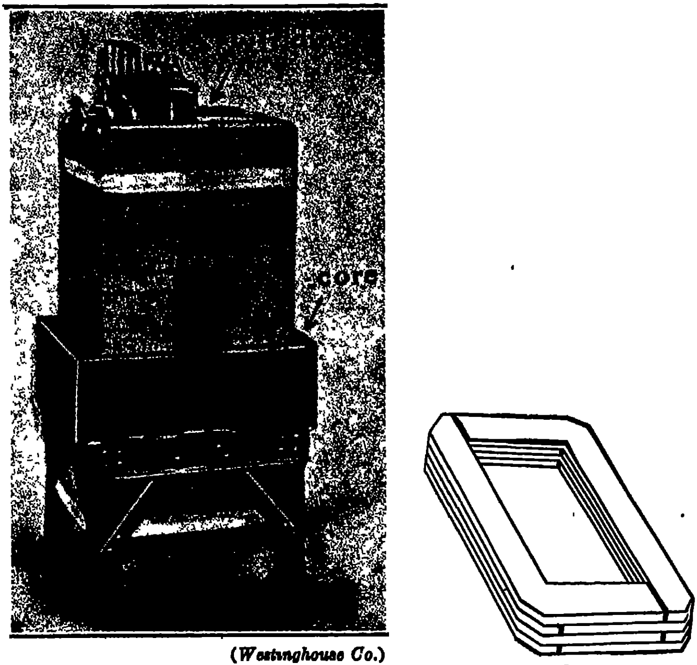
(*a*) (*b*)

Fig. 22.—High-voltage Condenser Bushing

$$\mathcal{R} = \frac{1}{\mu}\,\frac{l}{A}$$

where μ is the permeability of the material, l is the length of the magnetic circuit, and A is the section area of the magnetic circuit. In most transformers the section area of the magnetic circuit is not uniform and the calculation of the reluctance may be very difficult.

Figure 23*a* shows how the magnetic circuit is constructed around the winding. In this particular case the laminations consist of *L*-shaped stampings of silicon steel so arranged that excellent magnetic joints are obtained (see Fig. 23*b*).



(*Westinghouse Co.*)

(*a*) (*b*)

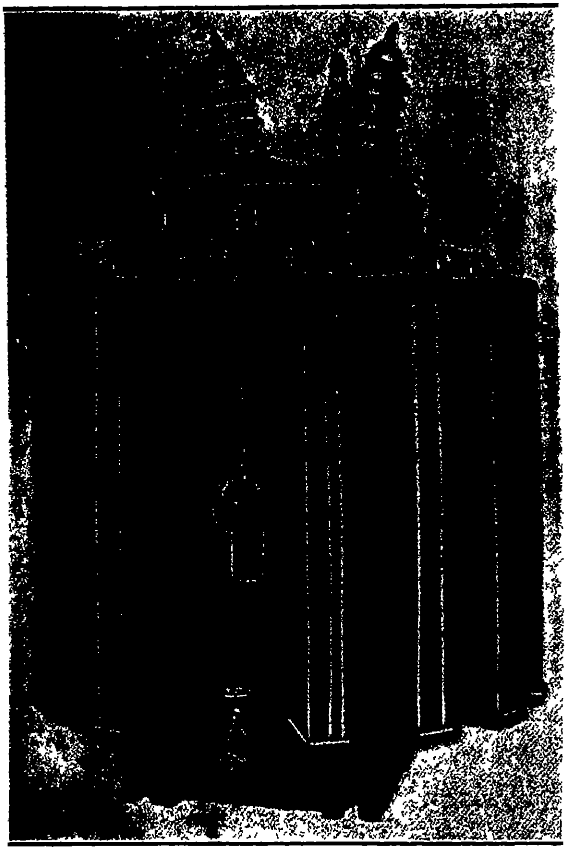
Fig. 23.—Partly Assembled Transformer and Illustration of the Method of Assembling the Core

32. Transformer Cooling.—The losses in the windings and in the magnetic circuit of a given transformer are very small, but on account of its compactness the losses are usually sufficient to raise the temperature beyond the safe operating limit unless special provision is made for keeping the transformer cool.

Transformers may be classified as to the following methods of cooling:

1. Natural radiation, conduction, and convection from the transformer to the surrounding medium.

2. Air-cooled (air is forced through the transformer winding and core by a blower).



(By the courtesy of the Westinghouse Co.)

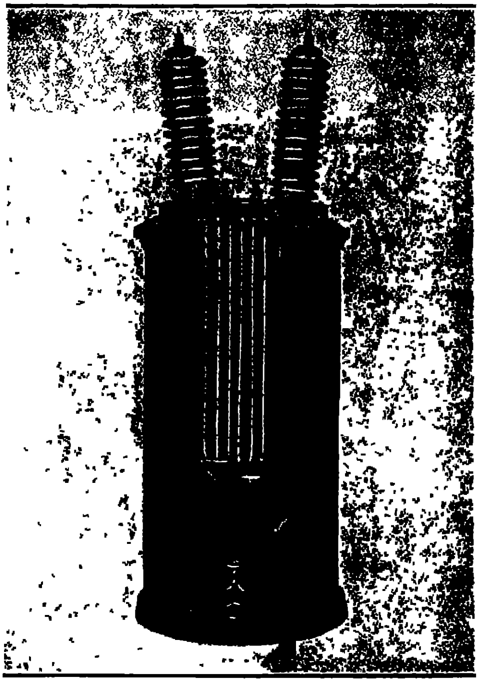
FIG. 24.—Transformer Showing External Radiators for Cooling

3. Oil-insulated self-cooled. (Fig. 24 represents an oil-insulated external radiator type of transformer. The oil circulates through the transformer in an upward direction and is cooled as it circulates through the external radiators.)

4. Oil-insulated water-cooled. (Fig. 25 represents an oil-insulated water-cooled transformer. The oil circulates through

the transformer in an upward direction and is cooled as it in contact with pipes through which cold water is circula an external pump.)

The natural method of cooling (method 1) is used wher amount of power transformed is small. Low-voltage curren potential transformers are cooled this way. Method 2 is where the amount of power is relatively large and the voltage relatively low. Methods 3 and 4 are used on most transformers and are particularly suited for insulating and cooling high-voltage high-capacity transformers.



(*By the courtesy of the Westinghouse Co.*)

Fig. 25.—Oil Insulated Water-cooled Transformer

33. Transformer Oil.—The oil which is used for cooling and insulating transformers is obtained by fractional distillation of petroleum. It must be free from moisture and destructive chemicals. The presence of a very small quantity of moisture absolutely ruins its insulating qualities and for this reason both the transformer windings and the oil must be dried before the oil is poured over the windings. The transformer is dried in an oven and the insulation resistance of its windings is taken as an indication of the degree of dryness. The oil is dried by forcing it through a filter.

In ordinary operation the oil is kept dry by (1) making the transformer tank air-tight, or (2) by using a " breather." Small tanks are readily made air-tight, but large ones are not and are usually provided with breathers.

Breathers permit the free expansion and contraction of the air above the oil in the transformer tank and keep the air dry. One form of breather consists of a box containing calcium chloride through which the air must circulate before it can enter the transformer tank.

CHAPTER III

THEORY OF THE TRANSFORMER

34. Principle of Operation of a Transformer.—An alternating-voltage impressed across the terminals of the primary winding of a transformer sets up an alternating-current in the winding and produces an alternating magnetic flux in the core. The major portion of the flux links both the primary and the secondary windings, and on account of its variation e.m.f.'s are induced in both windings. The e.m.f. in the primary winding is a counter voltage and limits the current in this winding. The e.m.f. in the secondary winding sets up the secondary load current. Since the power enters the primary winding and leaves the secondary winding a transformer may be considered as a sort of motor-generator in which the primary winding corresponds to a motor and the secondary winding to a generator.

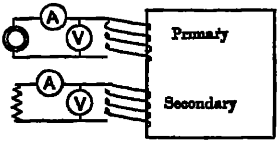


FIG. 26.—Arrangement of Meters for Measuring the Voltage and Current Relations of a Transformer

35. Approximate Voltage and Current Relations of a Power Transformer.—Fig. 26 shows the proper way for measuring the voltage and current relations in a power transformer. In the discussion which follows it is assumed that:

(1) The resistances of the windings on the transformer are very small.

(2) The flux linking the primary winding is at all instants equal to the flux linking the secondary winding.

(3) The reluctance of the magnetic circuit is small.

(4) The iron losses are small.

(5) The voltages, currents, and flux are sinusoids.

Approximate voltage ratio.—The voltage drop across the primary winding of a transformer is at all instants equal to the sum of the counter voltage drop due to the varying flux linking the winding and the voltage drop due to the resistance of the winding. That is

$$v_1 = e_1 + r'_1 i_1 \qquad (1)$$

where v_1 is the impressed voltage, e_1 is the counter voltage drop due to the flux and $r'_1 i_1$ is the resistance drop due to the primary current (the resistance r'_1 is the resistance of the winding to a direct-current and is often called the *ohmic resistance*).

In a power transformer the drop due to the resistance of a given winding is small; the windings are purposely designed this way so as to obtain a small copper loss. If it is assumed that the resistance drop is negligible the relation for the impressed voltage becomes

$$v_1 = e_1 \quad \text{(approximately)} \qquad (2)$$

That is, the impressed voltage and the counter voltage are approximately equal at all instants and consequently their wave shapes must be nearly the same.

If the flux links all the turns of both windings the induced voltages in the windings may be expressed by the relations

$$e_1 = N_1\left(\frac{d\phi}{dt}\right) \qquad (3)$$

and

$$e_2 = N_2\left(\frac{d\phi}{dt}\right) \qquad (4)$$

where e_1 and e_2 represent the primary and secondary voltages which are induced by the flux ϕ, N_1 represents the number of turns on the primary winding and N_2 the number of turns on the secondary winding.

By comparing equations (2) and (3) it is apparent that a sinusoidal voltage impressed on the primary winding of a transformer produces a sinusoidal flux which in turn produces sinusoidal electromotive forces in the primary and the secondary windings.

Since the secondary winding of a transformer corresponds to a generator it follows that the instantaneous voltage across the terminals of the secondary winding may be expressed by the relation

$$v_2 = e_2 - r'_2 i_2 \qquad (5)$$

Here again the voltage drop due to the resistance may be neglected and then

$$v_2 = e_2 \quad \text{(approximately)} \tag{6}$$

From equations (2), (3), (4), and (6) it follows that

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = \frac{v_1}{v_2} \quad \text{(approximately)} \tag{7}$$

The instantaneous voltages across the windings have the same ratio as the number of turns on the windings and the r.m.s. voltages must have the same ratio. That is

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{(approximately)} \tag{8}$$

Current ratio.—A transformer is so designed that the losses in its windings and core are very small. For this reason the power input to the primary winding is practically equal to the power output of the secondary winding. That is

$$V_1 I_1 \cos \theta_1 = V_2 I_2 \cos \theta_2 \quad \text{(approximately)} \tag{9}$$

The energy stored in a magnetic circuit is proportional to its reluctance and the square of the magnetic flux. Due to the low reluctance of the core of a transformer, the energy stored in its magnetic circuit is always small and for this reason the rate at which energy enters the magnetic circuit of a transformer is at all instants nearly equal to the rate at which energy leaves the magnetic circuit. The only way this condition can be satisfied is that the phase angle between the primary voltage and current be equal to the phase angle between the secondary voltage and current.

That is

$$\theta_1 = \theta_2 \quad \text{(approximately)} \tag{10}$$

or

$$\cos \theta_1 = \cos \theta_2 \quad \text{(approximately)} \tag{11}$$

From relations (8), (9), and (11), it follows that

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \quad \text{(approximately)} \tag{12}$$

The ampere-turns of the primary winding are approximately equal to the ampere-turns of the secondary winding.

The accuracy of equations (8) and (12) for the voltage and current ratios of a transformer depend on the load connected to the transformers. At no-load the only current flowing is a small current in the primary winding required to excite the core. The voltage drop due to this current is very small with the result that relation (8) is very nearly correct. On the other hand, at no-load, the current in the secondary winding is zero and relation (12) is entirely wrong. If the secondary winding is operated at over-load the currents in the windings produce appreciable voltage drops, relation (8) is no longer true, but relation (12) is approximately true. For exact calculations of the voltage and current relations it is necessary that a more complete analysis of the performance of a transformer be made.

36. E.M.F. Induced in a Winding on a Transformer and its Phase Relation to the Flux.—A sinusoidal flux of the form

$$\phi = \phi_{\max} \sin \omega t \qquad (13)$$

linking N turns of a winding on a transformer induces a voltage in the winding of the form

$$e = -N\frac{d\phi}{dt}10^{-8} = -\omega N\phi_{\max.} \cos \omega t 10^{-8} \qquad (14)$$

where e is the e.m.f. in the winding taken positive in the same sense (around the core) in which the primary current is taken as positive.

The maximum value of this e.m.f. is

$$E_{\max.} = \omega N\phi_{\max.}10^{-8} \qquad (15)$$

and its r.m.s. value is

$$E = \frac{2\pi fN\phi_{\max.}}{\sqrt{2}}10^{-8} = 4.44Nf\phi_{\max}10^{-8} \qquad (16)$$

The phase relation between the e.m.f. and the flux is given by equations (13) and (14). One of these expressions is a sine function and the other a cosine function with a minus sign in front of it. This proves that the flux and the e.m.f. are 90 time degrees apart and that the flux leads the e.m.f.

37. Voltage Current and Flux Relations in the Primary Winding of a Transformer Operating at No-load.—

Let V_1 = r.m.s. value of the voltage applied to the primary winding;
I_N = r.m.s. value of no-load current;
E = r.m.s. value of the counter voltage drop due to all the flux linking the primary winding;
and r_1' = ohmic resistance of the primary winding.

The relations between these quantities is

$$\underset{\cdot}{V}_1 = E + r'_1\underset{\cdot}{I}_N \quad \text{(sine-waves assumed)} \qquad (17)$$

This is the fundamental voltage-current relation for any receiver of power. Stated in words, the voltage drop across a receiver of power is equal to the vector sum of the counter voltage drop due to all the flux linking the circuit and the resistance drop.

If sinusoidal voltages and currents are assumed the power input to a transformer, at no-load, is

$$P_1 = V_1 I_N \cos\theta_N \qquad (18)$$

or

$$P_1 = E I_N \cos\theta'_N + r'_1 I^2_N \qquad (19)$$

where θ_N is the phase angle between V_1 and I_N and θ'_N is the phase angle between E and I_N.

The power $r'_1 I_N^2$ represents the copper loss in the winding and therefore the power $E I_N \cos\theta'_N$ must represent the core loss.

Since E, I_N, and the core loss are not zero, it follows that $\cos\theta'_N$ is not zero and that the angle between E and I_N must be less than 90 degrees. Since the e.m.f. and the flux [see equations (13) and (14)] are 90 degrees out of phase, it follows that the no-load current and the flux are not in phase.

38. Mutual Flux and Leakage Flux.—Frequently it is desirable to separate the total flux linking the primary winding of a transformer into two components. One of the components is called the mutual flux and the other the leakage flux. The mutual flux ϕ_M links all the turns of both the primary and secondary windings and the leakage flux ϕ_{L1} links the primary winding only. Similarly the flux linking the secondary winding may be separated into the components ϕ_M and ϕ_{L2}. On this basis the total flux linking the primary winding is

$$\underset{\cdot}{\phi}_1 = \underset{\cdot}{\phi}_M + \underset{\cdot}{\phi}_{L1} \qquad (20)$$

and the total flux linking the secondary winding is

$$\phi_2 = \phi_M - \phi_{L2} \quad \text{(see Fig. 30)} \tag{21}$$

39. Voltage, Current, Flux Diagram for the Primary Winding of a Transformer Operating at No-load (see Fig. 27).—

Let E_1 = counter voltage drop in the primary winding due to the mutual flux;

and E_{L1} = counter voltage drop in primary winding due to leakage flux;

then

$$V_1 = E_1 + E_{L1} + r_1' I_N \tag{22}$$

and

$$P_1 = V_1 I_N \cos\theta_N$$

$$= E_1 I_N \cos\theta''_N + E_{L1} I_N \cos\theta'''_N + r'_1 I_N^2 \tag{23}$$

where θ''_N is the phase angle between E_1 and I_N and θ'''_N is the phase angle between E_{L1} and I_N.

On account of the core losses due to the mutual flux and the leakage flux the angles θ''_N and θ'''_N are each less than 90 degrees.

Summary.—(1) The no-load current of a transformer is not in phase with the flux it produces. On account of the iron losses the current leads the flux.

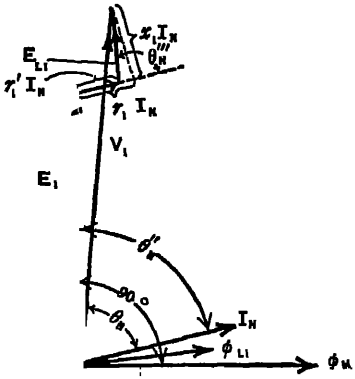


FIG. 27.—Vector Diagram for the Primary Winding of a Transformer Operating at No-load

(2) The e.m.f. induced by the flux lags the flux by 90 degrees (the e.m.f. is taken positive in the same sense as the current which produces the flux).

(3) The voltage drop due to an e.m.f. is in phase opposition to the e.m.f. (both taken positive in the same sense).

(4) The counter voltage drop due to the mutual flux is less than 90 degrees ahead of the no-load current.

(5) The counter voltage drop due to the leakage flux is less than 90 degrees ahead of the current.

40. Leakage Reactance and Effective Resistance.—The voltage drop E_{L1} (Fig. 27) is usually resolved into two components. One of these components is 90 degrees ahead of the current and is called the leakage reactance drop x_1I_N. The other component is in phase with the current; it is usually combined with the ohmic resistance drop r'_1I_N and the sum of these drops is called the effective resistance drop r_1I_N.

In general the effective resistance of a transformer winding may be expressed by the relation

$$r_1=r_1'+\frac{\left\{\begin{matrix}\text{core loss due to leakage flux}+\text{copper}\\ \text{loss due to flux cutting the winding}\end{matrix}\right\}}{(\text{current through winding})^2}$$

Roughly

$$r_1=1.15r'_1 \tag{24}$$

This relation holds for transformer windings only and is an approximation at that.

41. Primary Current Due to the Secondary Load (see Fig. 28).—The current in the primary winding of a transformer may be resolved into two components. One of the components is required to produce the core flux and is nearly constant. The other component depends on the secondary load and changes instantly when the secondary load is changed.

The component of the primary current due to the secondary load may be calculated on the basis that the total power entering the secondary winding may be represented by an equal power leaving the primary winding. That is

$$E_2I_2\cos\theta_2=E_1I_{21}\cos\theta_{21} \tag{25}$$

where E_2 = r.m.s. value of the e.m.f. in the secondary winding due to the mutual flux;

I_2 = r.m.s. value of the secondary load current;

θ_2 = phase angle between E_2 and I_2;

E_1 = voltage drop in the primary due to the counter voltage produced by mutual flux;

I_{21} = component of primary current due to secondary load;

and θ_{21} = phase angle between E_1 and I_{21}.

Since none of the power indicated by equation (25) is stored in the magnetic circuit, the phase angle between E_2 and I_2 must be the same as the phase angle between E_1 and I_{21}. That is

$$\cos\theta_2=\cos\theta_{21} \tag{26}$$

Since

$$\frac{E_2}{E_1}=\frac{N_2}{N_1}\quad\text{(exactly)} \tag{27}$$

it follows that

$$I_{21}=\frac{N_2}{N_1}I_2 \quad \text{(exactly)} \tag{28}$$

The component of the primary current due to the secondary load is equal to the product of the ratio of the secondary to the primary turns and the secondary current.

The total primary current is the vector sum of I_{21} (primary current due to the secondary load) and I_N (primary current necessary to produce the mutual flux). That is

$$\underset{\cdot}{I}_1=\underset{\cdot}{I}_{21}+\underset{\cdot}{I}_N \tag{29}$$

Figure 28 is the complete voltage drop-current vector diagram for the primary winding of a transformer when the secondary winding supplies an inductive load.

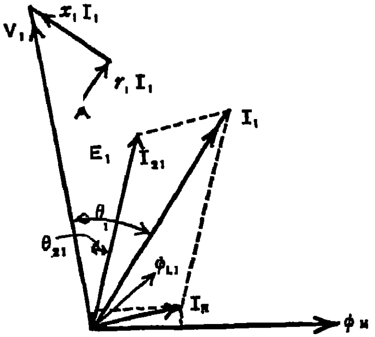


FIG. 28.—Vector Diagram for the Primary Winding of a Transformer Operating at Normal Load

42. Exciting Current; Variation of Core Flux and Exciting Current with Load; Expressions for the Exciting Current; Error Made in Assuming Exciting Current a Sine-wave.—

Exciting current.—Equation (28) shows that the magnetizing action of the secondary winding results in an equal and opposite magnetizing action in the primary winding (the ampere-turns N_2I_2 tend to produce a core flux in one direction and the ampere-turns N_1I_{21} in the opposite direction). The vector difference between the total ampere-turns N_1I_1 of the primary winding and the ampere-turns N_1I_{21} due to secondary load must represent the ampere-turns which produce the magnetic flux in the core. That is

$$N_1\underset{\cdot}{I}_1-N_1\underset{\cdot}{I}_{21}=N_1\underset{\cdot}{I}_N \tag{30}$$

The current I_N is often called the exciting current and is equal to the no-load current in the primary winding.

Variation of core flux and exciting current with load.—Since the resistance and the leakage reactance of the primary winding of a transformer are usually very small, it follows that for all ordinary loads the voltage drops produced by these quantities are small and therefore the impressed voltage on the primary winding is approximately equal to the counter voltage produced by the mutual flux (see Fig. 28). On this basis when the impressed frequency and the r.m.s. value of the impressed voltage are constant, the r.m.s. value of the counter voltage is nearly constant, and the r.m.s. value of the mutual flux which is directly related to the counter voltage must remain practically constant. The power losses in the magnetic circuit depend on the mutual flux and are therefore practically constant provided the impressed voltage and frequency are constant.

The exciting current may be considered as having two components, both of which depend on some function of the impressed voltage and frequency. The component of the exciting current which is in phase with the counter-voltage drop (due to the mutual flux) is necessary to supply the iron losses due to the mutual flux and the component which lags the counter voltage by 90 degrees is necessary to supply the reactive power in the magnetic circuit.

Mathematical expressions for the exciting current.—The component of the exciting current which is in phase with the counter-voltage drop is called the core loss (hysteresis+eddy-current) component. By definition this component is equal to the core-loss conductance multiplied by the counter voltage. That is

$$I_{h+e}=g_1E_1 \tag{31}$$

The core-loss conductance may be calculated by the relation

$$g_1=\frac{\text{Core loss}}{E_1^2} \tag{32}$$

The component of the exciting current which lags the counter-voltage drop by 90 degrees is in phase with the mutual flux and is called the magnetizing component. The magnitude of this current is equal to the product of the magnetizing susceptance and the counter-voltage drop. That is

$$I_m=b_1E_1 \tag{33}$$

The magnetizing susceptance may be calculated by the relation

$$b_1=\frac{\text{Reactive power in the magnetic field}}{E_1^2} \tag{34}$$

The expression for the exciting current in terms of its components is

$$I_N = \underset{\cdot}{I}_{h+e} + \underset{\cdot}{I}_m$$

or

$$\underset{\cdot}{I}_N = (g_1 - jb_1)\underset{\cdot}{E}_1 \tag{35}$$

Wave shape of the exciting current of a transformer.—It was demonstrated in Article 35 that a sinusoidal voltage when impressed across the primary winding of a transformer results in a sinusoidal core flux. It will now be demonstrated that a sinusodial flux in an iron core must be produced by a non-sinusoidal current.

Figure 29*a* represents the hysteresis loop of the iron used in a certain transformer and Fig. 29*b* represents the (hysteresis+magnetizing) component of the current obtained from the loop in a graphical way. For example, the current required to produce the flux PQ is equal to $OM = QR$; the current required to produce the flux ST is equal to $ON = SU$; etc. The current wave obtained is not a sine-wave; the flux and the current have their maximum values at the same instant, but pass through zero values at entirely different instants.

The wave shape of the exciting current depends on the wave shapes of the *hysteresis+magnetizing* component and on the component which represents the eddy-current loss in the core. Since the latter component represents a true power loss, and nothing else, it must be a sine-wave (see Article 24). A sine-wave quantity combined with a non-sinusoidal quantity results in a non-sinusoidal quantity. For these reasons the exciting current of a power transformer is usually a non-sinusoidal quantity.

Error made in assuming the exciting current a sine-wave.—An analysis of the current wave in Fig. 29*b* shows that an appreciable third harmonic is present and the problem arises as to how the harmonics must be taken into account in circuit calculations. In certain transformer connections (see Articles 58 and 131) harmonics produce undesirable results and for this reason an analysis of a circuit involving a special connection requires that the harmonics be taken into account. The effect of a harmonic depends on the kind of circuit and on the value of the harmonic.

If no special connections are involved the error made in assuming the exciting current a sine-wave may be readily calculated.

For example, if an analysis of the current wave shows that the fundamental has an r.m.s. value of 5 amperes and that the third harmonic is 20 per cent of the fundamental, the true r.m.s. value of the current is $\sqrt{5^2+1^2}=5.1$ amperes (see Article 25). Thus, in assuming that this current is a sine-wave having an r.m.s. value of 5.1 amperes involves an error of only 2 per cent. Again when the primary voltage is a sine-wave the total power input due to the exciting current depends on the fundamental in the exciting current and is independent of the higher harmonics. In this particular case only a small error is made in basing power calculations on a fundamental of 5.1 amperes instead of 5 amperes. For these reasons exciting currents are ordinarily treated as sine-waves and may be represented by vectors on vector diagrams.

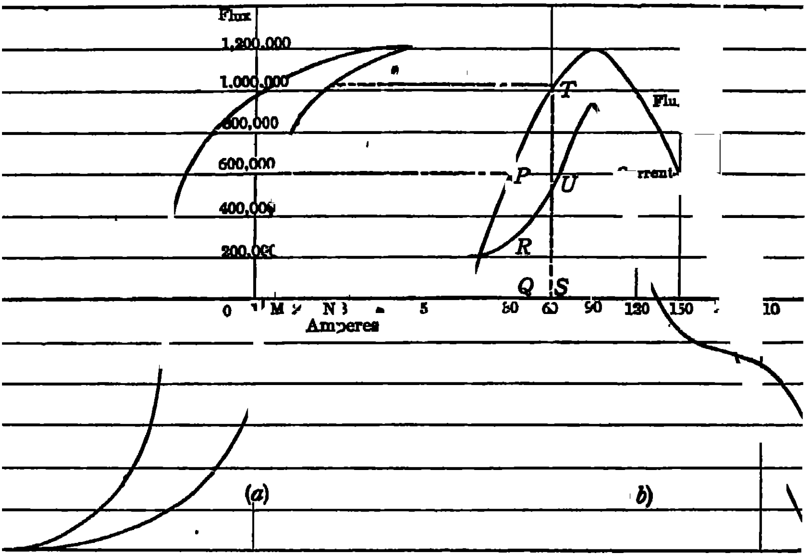


Fig. 29.—Method for Determining the Wave Shape of the Magnetizing + Hysteresis Component of the No-load Current of a Transformer

43. Complete Voltage-Current-Flux Diagram for a Transformer and the Fundamental Voltage Current and Power Relations.—The position of vectors on vector diagrams depends partly on the positive sense chosen. In Fig. 30 the primary voltage drop and current are assumed positive in the sense *a* to *b* and the secondary voltage rise and current are assumed positive in the sense *c* to *d*.

Summary of voltage current and power relations (see Fig. 30).—

Primary voltage.—

$$\underset{\cdot}{V}_1 = \underset{\cdot}{E}_1 + (r_1 + jx_1)\underset{\cdot}{I}_1 \qquad (36)$$

Primary current.—

$$\underset{\cdot}{I}_1 = \underset{\cdot}{I}_{21} + \underset{\cdot}{I}_N \qquad (37)$$

Primary power input.—

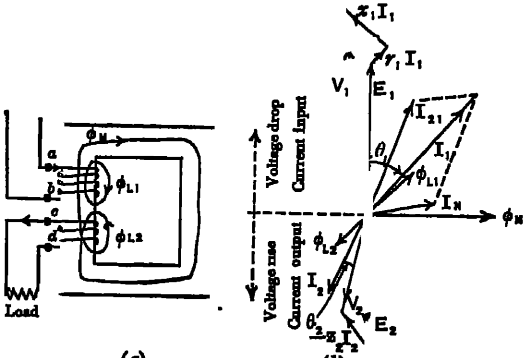
$$P_1 = V_1 I_1 \cos \theta_1 \qquad (38)$$

Secondary voltage.—

$$\underset{\cdot}{V}_2 = \underset{\cdot}{E}_2 - (r_2 + jx_2)\underset{\cdot}{I}_2 \qquad (39)$$

Secondary power output.—

$$P_2 = V_2 I_2 \cos \theta_2 \qquad (40)$$



(a) (b)

FIG. 30.—Complete Vector Diagram for the Primary and Secondary Windings of a Transformer

In these relations E_1 and E_2 are the primary and secondary voltages due to the mutual flux; r_1 and r_2 are the effective resistances of the windings; x_1 and x_2 are the leakage reactances of the windings due to the leakage fluxes ϕ_{L1} and ϕ_{L2}; I_{21} is the primary current due to the secondary load; I_N is the no-load current; θ_1 is the phase angle between V_1 and I_1; and θ_2 is the phase angle between V_2 and I_2.

44. Exactly Equivalent Circuit of a Transformer.—In order to predetermine the performance of a transformer it is advisable to base all calculations on an equivalent circuit. A simple circuit which when connected to the same supply as an actual transformer takes the same current at the same power factor as the transformer is said to be equivalent to the transformer.

Figures 31*a*–*e* show how the exactly equivalent circuit is derived. The equivalent circuit may be referred to either the high-voltage or the low-voltage sides. The circuit shown in Fig. 31*e* is referred to the high-voltage side.

Figure 31*a* represents the actual transformer of which the equivalent circuit is desired. The subscripts H and L are used to designate the high- and low-voltage windings. The expression LH means low-voltage side referred to high-voltage side.

The first step in obtaining the required circuit is to replace the windings on the transformer by ideal windings (windings having no resistance and no leakage reactance). In Fig. 31*b* the windings are perfect, but resistances and reactances equal to those of the windings of the actual transformer are connected in the leads of this fictitious transformer. It should be apparent that the circuits of this transformer are equivalent to those on the actual transformer.

Figure 31*c* represents a transformer having ideal windings and a turn ratio of unity. The primary circuit of this transformer is equivalent to the primary circuit of the actual transformer provided the magnetic reactions due to the secondary currents are equal and bear the same time phase relations to the respective impressed voltages.

If we assume that the load on the actual transformer is connected to the low-voltage winding and that the outputs and load power factors of the actual transformer and fictitious transformer are respectively equal, we obtain the relations

$$V_{LH}I_{LH}\cos\theta_{LH} = V_LI_L\cos\theta_L \tag{41}$$

and

$$V_{LH}I_{LH} = V_LI_L \tag{42}$$

For equal secondary winding magnetic reactions it is necessary that

$$N_HI_{LH} = N_LI_L$$

or

$$I_{LH} = \frac{I_L}{U} \tag{43}$$

where U is the turn ratio.

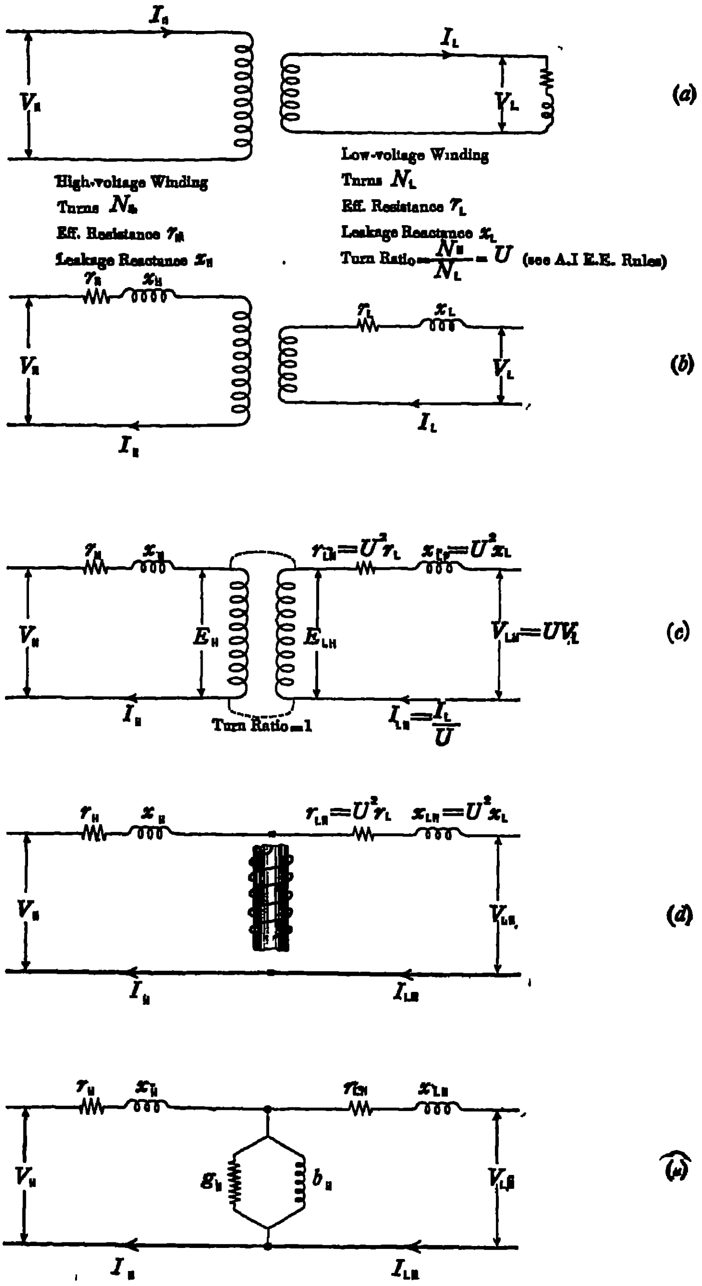


Fig. 31.—Exactly Equivalent Circuit of a Transformer Referred to the High-voltage Side

Substituting (43) in (42), we obtain

$$V_{LH} = UV_L \tag{44}$$

For equal copper losses in the secondary windings it is necessary that

$$r_{LH}I^2_{LH} = r_LI^2_L \tag{45}$$

Substituting (43) in (45), we obtain

$$r_{LH} = U^2r_L \tag{46}$$

In order to obtain the same internal secondary winding power factor it is necessary that the reactances bear the same relation as the resistances. That is

$$x_{LH} = U^2x_L \tag{47}$$

If relations (41) to (47) are all satisfied at once it follows that:

1. The actual power outputs of the real and fictitious transformers are equal.
2. The secondary winding copper losses are equal in the two transformers.
3. The ampere-turns of the secondary windings are equal.
4. The internal power factors of the secondary windings are equal. That is

$$\cos \theta \frac{E_{LH}}{I_{LH}} = \cos \theta \frac{E_L}{I_L}$$

and from 3 and 4 it follows that:

5. The magnetic reactions of the secondary windings are equal and these reactions bear the same time phase relations to the primary impressed voltages.

The primary and secondary winding may now be connected as shown by the dotted connection in Fig. 31*c* (the + terminal of the primary winding is connected to the + terminal of the secondary winding). No current flows as a result of this connection for $E_{LH} = E_H$.

The two windings in Fig. 31*c* may now be replaced by a single perfect winding having N_H turns (Fig. 31*d*) wound on an iron core, and this winding may be replaced by a parallel circuit, one branch a perfect resistance having a conductance g_H (the core-loss conductance) and the other branch a perfect inductance having a susceptance b_H (the magnetizing susceptance). The final circuit (Fig. 31*e*) is not a transformer, but it is an equivalent circuit of the actual transformer.

Voltage drop-current vector diagram of the exactly equivalent circuit of a transformer (see Figs. 31*e* and 32).—The student should compare this vector diagram with that in Fig. 30. The current I_H corresponds to I_1; the current I_{LH} corresponds to the current I_{21}; the current I_N corresponds to the current I_N; the counter-voltage drop E_H corresponds to E_1; and the voltage drop V_H corresponds to V_1.

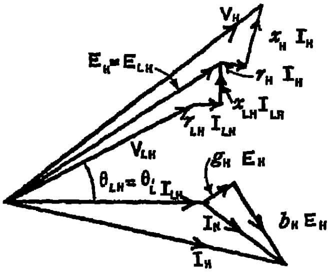


FIG. 32.—Vector Diagram for the Exactly Equivalent Circuit of a Transformer

45. Approximately Equivalent Circuit of a Transformer Referred to the High-voltage Side.—Only a small error is made in changing the location of the exciting circuit from the position shown in Fig. 31*e* to that shown in Fig. 33.

Vector diagram for the approximately equivalent circuit of a transformer (see Figs. 33 and 34).—The vector diagram is based on the approximate relations

$$\underset{.}{V}_H = \underset{.}{V}_{LH} + (R_H + jX_H)\underset{.}{I}_{LH}$$

and

$$\underset{.}{I}_N = (g_H - jb_H)\underset{.}{V}_H$$

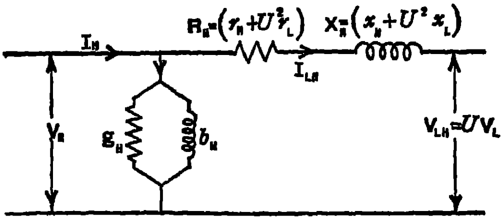


FIG. 33.—Approximately Equivalent Circuit of a Transformer Referred to the High-voltage Side

46. Voltage Regulation and Efficiency of a Transformer.—In the following discussion the primary winding of a transformer will be designated by the subscript 1 and the secondary winding by the subscript 2. The double subscript 12 means primary referred to the secondary and the subscript 21 means secondary referred to primary.

Voltage regulation.—The voltage regulation of a transformer is the ratio of the difference between the no-load and the rated load terminal voltage of the secondary winding to the rated load terminal voltage. The per cent regulation may be determined by direct test or may be calculated by using the circuit constants. In either case the regulation may be calculated by the relation

$$\text{Per cent regulation} = \frac{V_1 - V_{21}}{V_{21}}100 = \frac{V_{12} - V_2}{V_2}100 \qquad (48)$$

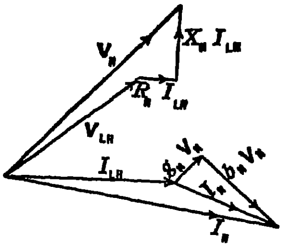


FIG. 34.—Vector Diagram for the Approximately Equivalent Circuit of a Transformer

where V_2 is the rated secondary terminal voltage, V_{21} is the rated secondary terminal voltage referred to the primary, V_1 is the primary terminal voltage, and V_{12} is the primary terminal voltage referred to the secondary side.

If the approximately equivalent circuit of a transformer is assumed the following method for calculating the regulation may be employed (see A.I.E.E. Rules):

$$\text{Per cent reg} = mq_r + nq_x + \frac{(mq_x - nq_r)^2}{200} \qquad (49)$$

where $q_r = \frac{R_2 I_2}{V_2}100 = \frac{R_1 I_{21}}{V_{21}}100$ (this is called the per cent resistance drop)

$q_x = \frac{X_2 I_2}{V_2}100 = \frac{X_1 I_{21}}{V_{21}}100$ (this is called the per cent reactance drop)

$m = \cos\theta_2 = \cos\theta_{21}$ (power factor of load)

$n = \sin\theta_2 = \sin\theta_{21}$ (reactive factor of load)

$R_1 = r_1 + r_{21}$ (equivalent winding resistance)

$X_1 = x_1 + x_{21}$ (equivalent winding leakage reactance).

47. Efficiency.—In general the efficiency of any machine is the ratio of the useful power output to the total power input. For any given load on a transformer the mathematical expression for the efficiency is

$$\text{Per cent efficiency at a given load} = \frac{\text{Output}}{\text{Output}+\text{copper loss}+\text{core loss}}100 \qquad (50)$$

When the load on a transformer varies considerably throughout a day it is sometimes desirable to obtain its all-day efficiency. This efficiency may be calculated by the relation

$$\text{Per cent all-day efficiency} = \frac{\text{Output (for 24 hrs.)}}{\text{Output (for 24 hrs.)}+\text{losses (for 24 hrs.)}}100 \qquad (51)$$

48. Method for Measuring the Effective Resistance, Leakage Impedance, and Leakage Reactance of a Transformer (Impedance Test).—The effective resistance and leakage reactance of a transformer winding may be closely approximated by calculation but cannot be measured by a simple test. Fortunately in most problems on transformers it is necessary to know the resistances and reactances of both windings combined and not the resistance and reactance of each winding separately. The resistance and reactance of both windings combined may be measured by a simple test known as the "impedance or load-loss test" (see A.I.E.E. Rules).

In the impedance test the low-voltage winding is short-circuited (see Fig. 35) and the high-voltage winding is connected through a protecting resistance to a source of a-c. power which operates at the rated frequency of the transformer. The voltage applied to the high-voltage winding must be small but sufficient to circulate rated current through both windings. Meter readings of the voltage, current, and power are taken.

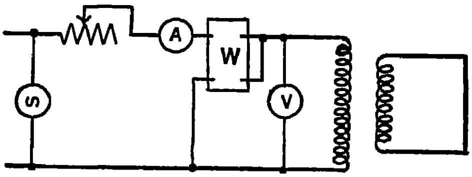


Fig. 35.—Arrangement of Meters for Impedance or Load-loss Test on a Transformer

Since the voltage across the primary winding is small and since the core loss depends approximately of the square of this voltage it may be assumed that the core loss is negligible and that the total power input to the transformer under short-circuited conditions is equal to the copper losses in both windings. On this basis the effective resistances of both windings may be calculated by the relation

$$R=\frac{W}{I^2} \tag{52}$$

where R is the effective resistance of both windings referred to the side in which the meters are connected, W is the power indicated by the wattmeter, and I the current indicated by the ammeter.

The leakage impedance of both windings combined referred to the side in which the meters are connected is

$$Z=\frac{V}{I} \tag{53}$$

and the leakage reactance of both windings combined referred to the same side is

$$X=\sqrt{Z^2-R^2} \tag{54}$$

Since the constants R and Z of a transformer depend on the temperature it is necessary when making the impedance test to observe the temperature of the windings. If the temperature of the windings is $t°$ C. and it is desired to know the constants at the standard reference temperature of 75° C. the following calculations must be made:

1. The effective resistance

$$R_{75}=R_t[1+\alpha^t(75-t)] \quad \text{approximately} \tag{55}$$

where α^t is the temperature coefficient of resistivity at the temperature t.

2. The leakage reactance

$$X_{75}=X_t \quad \text{approximately} \tag{56}$$

and

3. The leakage impedance

$$Z_{75}=\sqrt{(R_{75})^2+(X_t)^2} \quad \text{approximately} \tag{57}$$

The first of these relations is an approximation for the reason that the effective resistance may be resolved into a part representing the ohmic resistance of the winding and a part which when multiplied by the square of the current represents the core loss due to the leakage flux. The part representing the ohmic resistance of the winding should be corrected for temperature but the part which represents core loss should not be corrected for temperature. Since the reluctance of the path of the leakage flux at ordinary temperatures is practically independent of the temperature it is safe to say that the second relation is nearly true. The third relation is an approximation for the simple reason that it depends on two relations, one of which is an approximation.

49. Method for Determining the Exciting Current, Core-loss Conductance, and Magnetizing Susceptance (No-load Test).—The magnitude of the exciting current and its phase relation to the impressed voltage are determined by the no-load test (see A.I.E.E. Rules). The transformer is connected as shown in Fig. 36 to a source of a-c. power which must be operated at the rated frequency of the transformer. In this test low-range meters must be used and it is very important that the current coils of the ammeter and wattmeter be protected by a suitable short-circuiting switch at the instant when the transformer is first connected to the supply. The rush of exciting current obtained when a transformer is connected to the supply depends on the state of magnetism in the core. If the core has appreciable residual magnetism it is possible that the first rush of exciting current is many times the rated current of the transformer and for this reason the meters must be protected.

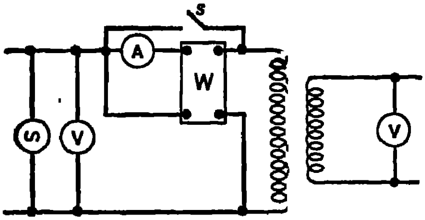


FIG. 36.—Arrangement of Meters for No-load Test on a Transformer

The voltage impressed on the primary winding must be sufficient to obtain across the secondary winding a voltage equal to the rated secondary voltage $+ r_2I_2$ (the resistance drop in the secondary winding at rated load). The reason for making the test at this voltage is to obtain approximately the same mutual flux at no-load as is obtained when the transformer is operated at rated unity power factor load.

The core-loss conductance referred to the side in which the ammeter and wattmeter are connected may be calculated from the relation

$$g=\frac{P}{E^2} \tag{58}$$

Where g is the conductance, P is the core loss due to the mutual flux and is equal to the wattmeter indication less the power consumed by secondary voltmeter less the rI^2 loss in the windings at no-load, and E is the counter voltage due to the mutual flux. Since the impressed voltage V and the counter voltage due to the mutual flux are nearly equal and since the wattmeter reading W is nearly equal to the core loss due to the mutual flux it follows that

$$g=\frac{W}{V^2} \quad \text{(approximately)} \tag{59}$$

The exciting admittance referred to the side in which the ammeter is connected may be calculated by the relation

$$Y=\frac{I_N}{V} \quad \text{(approximately)} \tag{60}$$

Knowing the admittance and conductance the magnetizing susceptance may be calculated by the relation

$$b=\sqrt{Y^2-g^2} \quad \text{(approximately)} \tag{61}$$

These constants are referred to the side of the transformer in which the ammeter, voltmeter, and wattmeter are connected (the high-voltage side in this case). For the sake of safety the no-load test is usually performed with the meters connected in the low-voltage side. The core-loss conductance on the high-voltage side may be calculated by multiplying the core-loss conductance on the low-voltage side by the factor $\frac{1}{U^2}$. Similarly the magnetizing susceptance on the high-voltage side is $\frac{1}{U^2}$ times the magnetizing susceptance on the low-voltage side.

The core-loss conductance and the magnetizing susceptance at ordinary operating temperatures are independent of the temperature and no correction for temperature is necessary.

50. Turn Ratio Test.—The turn ratio of a transformer may be obtained from the designer or may be approximately measured by determining the ratio of the indications of two voltmeters, one connected across the high-voltage side and the other connected across the low-voltage side. In the turn ratio test the transformer is operated at no-load.

51. Values for the Exciting Current, Resistance Drop, Reactance Drop, Efficiency, and Regulation of a 5000 KV-A., 140,000- to 5000-volt, 60-cycle Transformer.—

Core loss.................................. ...... =41,500 watts
Exciting current.............................. = 7.0 per cent
Resistance drop................ .... = 7.72
Reactance drop............. .... = 8.63
Efficiency at rated unity power factor load ...... =98.4
Voltage regulation at rated unity power factor load = 8.1

PROBLEMS

1. In a certain coil 200 turns are linked by an alternating flux having a maximum value of 10^6 lines. The flux is produced by a 60-cycle current. (*a*) Calculate the r.m.s. value of the e.m.f. induced in the winding. (*b*) Construct a vector diagram showing the phase relation between the flux and the e.m.f. (*c*) What assumptions did you make in parts *a* and *b*?

2. A certain reactor having an iron core is connected to a 220-volt 25-cycle supply. The total iron loss is 425 watts of which 205 watts are due to hysteresis. (*a*) Calculate the total iron loss when the frequency is kept constant and the voltage decreased 10 per cent. (*b*) Calculate the total iron loss when the voltage and the frequency are both increased 10 per cent. (*c*) What assumptions were made in *a* and *b*?

3. An iron core reactor consumes 1210 watts when connected to a 110-volt source of a-c. power. The current is 41 amperes. The ohmic resistance of the coil is 0.15 ohm. (*a*) How much power is transferred from the coil to the magnetic circuit? (*b*) What is the phase angle between the current and the flux? (Assume that the voltage, the current, and the flux are sinusoidal quantities and assume that the leakage flux is zero.) (*c*) Construct a complete voltage drop-current-flux vector diagram.

4. The following measurements were made on a 2-kv-a. transformer at no-load:

Primary Voltage	Frequency	Primary Current	Secondary Voltage
110	60	0.57	440

(*a*) Calculate the mutual inductance between the primary and secondary windings. (*b*) What assumptions were made in (*a*)?

5. (*a*) Plot the hysteresis loop between flux as ordinates and current as abscissas given the following values for one half of the loop:

i	$\phi/10^6$	i	$\phi/10^6$
+1.15	0	+3.0	+1.165
+1.5	+0.62	+2.0	+1.1
+2.0	+0.85	+1.0	+1.00
+3.0	+1.07	+0.0	+0.83
+4.0	+1.18	−0.5	+0.68
+4.7	+1.2	−1.0	+0.39
+4.0	+1.19	−1.15	0.00

(Assume that the top half and the bottom half of the loop have the same shape.)

(*b*) Determine the wave shape of the current necessary to produce a sinusoidally varying flux having a maximum value of 1.2×10^6 lines.

(*c*) Analyze the current wave obtained for the fundamental, the 3rd harmonic, and the 5th harmonic.

(*d*) Why is no appreciable error made in assuming this current a sine-wave?

6. The following things are known about a 2-kv-a. 60-cycle transformer:

No-load current, 0.57 ampere;
Core loss, 24 watts;
D-c. resistance of primary winding, 0.07 ohm;
Turns on the primary winding, 60;
Maximum mutual flux linking the primary and secondary windings, 700,000 lines (assumed to vary sinusoidally);
Maximum leakage flux linking primary winding, 20,000 lines (assumed to vary sinusoidally);
Power loss due to the leakage flux negligible.

(*a*) Calculate the r.m.s. value of the induced e.m.f. in the primary winding due to the change in linkages produced by the mutual flux.

(*b*) Calculate the r.m.s. value of the induced e.m.f. in the primary winding due to the leakage flux.

(*c*) Determine the r.m.s. value of the primary terminal voltage.

(*d*) Construct a complete voltage drop-current-flux vector diagram.

7. A step-down transformer has a turn ratio of 4. The primary voltage s 444 volts and is held constant. The load on the secondary requires 20 mperes at 70 volts. The proper voltage at the load may be obtained by onnecting a 2-ohm rheostat in the secondary leads or a 32-ohm rheostat in he primary leads. Show why this is true.

8. The following tests were made on a 2-kv-a. 110/440-volt 60-cycle, tep-up transformer:

No-load Test (Meters Connected in the Low-voltage Side)

Impressed Voltage	Current	Power	Frequency
110	0.57	24	60

Impedance or Load-loss Test (Meters Connected in the High-voltage Side)

Impressed Voltage	Current	Power	Frequency
19	4.55	50	60

Turn ratio, 4;
D-c. resistance of the high-voltage winding, 0.86 ohm;
D-c. resistance of low-voltage winding, 0.08 ohm;
Temperature at which the measurements were made 20° C.

Determine: (*a*) the ratio of the equivalent effective resistance to the equivalent d-c. resistance of the windings; (*b*) the constants of a circuit exactly equivalent to the transformer at 75° C. referred to the high-voltage side (assume that the leakage reactance of the low-voltage winding referred to the high-voltage winding is equal to the leakage reactance of the high-voltage winding); (*c*) the constants of a circuit approximately equivalent to the transformer at 75° C. referred to the low-voltage side.

9. Based on the exactly equivalent circuit in Problem 8, calculate the voltage regulation of the transformer at 75° C. for:

(*a*) Rated kv-a. unity power factor load,
(*b*) Rated kv-a. seventy per cent leading power factor load,
(*c*) Rated kv-a. seventy per cent lagging power factor load.

10. Based on the approximately equivalent circuit, Fig. 34, derive equation (49). (Note that products of the form q_r^2 or $q_x q_r$ are very small.)

11. Use equation (49) and calculate the voltage regulation of the transformer in Problem 8 for the loads specified in Problem 9.

CHAPTER IV

TRANSFORMER APPLICATIONS

52. Parallel Operation of Two Transformers.—When the load on a transformer exceeds its rating it becomes necessary to replace the transformer or to operate another transformer in parallel with its secondary windings. The chief difficulty in operating two transformers in parallel is to obtain the proper distribution of the load. Transformers having equal ratings and equal constants are readily operated in parallel for the proper distribution of the load is automatically accomplished. Transformers of unlike ratings and unlike constants may or may not give satisfactory parallel operation, depending on certain factors which will now be considered.

53. Parallel Operation of Transformers at No-load (see Fig. 37).—The purpose of this article is to show that certain transformers cannot be operated in parallel even at no-load. For example, only a small difference in the turn ratios of the transformers results in very large circulating currents through the windings. This follows from the fact that the leakage impedance of the mesh formed by the secondary windings is very small and consequently a small difference in the secondary e.m.f.'s results in a large secondary current and also a large primary current.

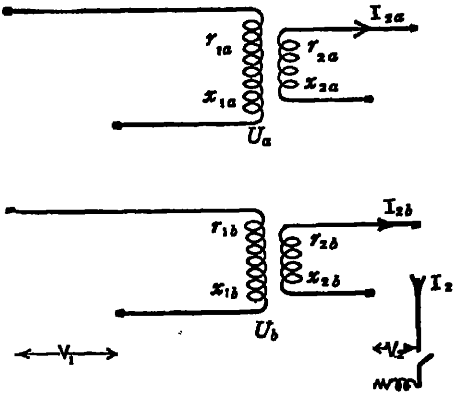


FIG. 37.—Parallel Operation of Transformers

In Fig. 37 the current in the mesh formed by the secondary windings (taken positive in the same sense as the e.m.f. E_{2a}) may be calculated by the relation

$$I_{2a} = -I_{2b} = \frac{E_{2a} - E_{2b}}{z_{2a} + z_{2b}} \tag{1}$$

where E_{2a}, E_{2b} are the e.m.f.'s induced in the secondary windings, and z_{2a}, z_{2b} are the leakage impedances of the secondary windings. The voltages E_{2a} and E_{2b} depend on the turn ratio, the primary leakage impedances, the primary impressed voltage. The value of the circulating current I_{2a} therefore depends on the leakage impedances of both the primary and the secondary windings of the transformers. Since the impedances of the windings are usually very small it follows that a small difference between the voltages E_{2a} and E_{2b} results in an excessive circulating current making parallel operation impossible. For this reason transformers whose turn ratios are only slightly different cannot be operated in parallel.

54. Parallel Operation of Transformers with the Secondary Load Connected.—In the following discussion the approximately equivalent circuits of the transformers are used and it is assumed that

$U_a = U_b =$ turn ratio;

$R_{2a} =$ effective resistance of transformer A referred to the secondary side;

$R_{2b} =$ effective resistance of transformer B referred to the secondary side;

$X_{2a} =$ leakage reactance of transformer A referred to the secondary side;

$X_{2b} =$ leakage reactance of transformer B referred to the secondary side;

$$\tan \beta_{2a} = \frac{X_{2a}}{R_{2a}};$$

$$\tan \beta_{2b} = \frac{X_{2b}}{R_{2b}};$$

and $V_{12} =$ primary voltage referred to the secondary side.

Assume that the transformers are connected correctly. (They must be connected that for the major portion of a cycle, the secondary voltages buck with reference to the mesh formed by the secondary windings.)

Since the transformers are so connected that the primary impressed voltages must be equal and the secondary load voltages must be equal it follows that for transformer *A*

$$\underset{\cdot}{V}_{12}=\underset{\cdot}{Z}_{2a}\underset{\cdot}{I}_{2a}+\underset{\cdot}{V}_{2} \tag{2}$$

and for transformer *B*

$$\underset{\cdot}{V}_{12}=\underset{\cdot}{Z}_{2b}\underset{\cdot}{I}_{2b}+\underset{\cdot}{V}_{2} \tag{3}$$

From equations (2) and (3) it follows that

$$\underset{\cdot}{Z}_{2a}\underset{\cdot}{I}_{2a}=\underset{\cdot}{Z}_{2b}\underset{\cdot}{I}_{2b} \tag{4}$$

or

$$Z_{2a}I_{2a}e^{j(\beta_{2a}+\theta_{2a})}=Z_{2b}I_{2b}e^{j(\beta_{2b}+\theta_{2b})} \tag{5}$$

$$\therefore\ \frac{I_{2a}}{I_{2b}}=\frac{Z_{2b}}{Z_{2a}} \tag{6}$$

The ratio of the secondary currents is equal to the inverse ratio of the leakage impedances of the transformers. Since the secondary voltages are equal it follows that the volt-ampere outputs of the transformers are inversely proportional to the leakage impedances of the transformers.

From equation (5)

$$\beta_{2a}+\theta_{2a}=\beta_{2b}+\theta_{2b} \tag{7}$$

and if

$$\beta_{2a}=\beta_{2b}$$

it follows that

$$\theta_{2a}=\theta_{2b} \tag{8}$$

That is when the impedance angles are equal the load power factor angles are also equal.

If the load power factors are equal it follows that

$$I_{2a}+I_{2b}\text{(arithmetical sum)}=I_2 \tag{9}$$

and that

$$\frac{I_{2a}}{I_{2b}}=\frac{R_{2b}+jX_{2b}}{R_{2a}+jX_{2a}}=\frac{R_{2b}}{R_{2a}}=\frac{X_{2b}}{X_{2a}} \tag{10}$$

The currents divide inversely as the resistances. This is the condition necessary for minimum copper loss, but is not necessarily the condition for equal temperature rise of the transformers. Equation (10) may always be satisfied by changing the *R*'s and *X*'s of the transformer leads.

55. Summary of Conditions Necessary for Satisfactory Operation of Two Transformers in Parallel.—

1. The transformers must be properly connected as to polarity.
2. The turn ratios must be equal.
3. For minimum copper loss

$$I_{2a}+I_{2b}(\text{arithmetical sum})=I_2$$

and

$$\frac{I_{2a}}{I_{2b}}=\frac{R_{2b}}{R_{2a}}$$

4. To utilize the full kv-a. capacities of the transformers the ratings of the transformers must be proportioned inversely as their impedances. That is

$$\frac{\text{Rated kv-a. of transformer } A}{\text{Rated kv-a. of transformer } B}=\frac{I_{2a}}{I_{2b}}=\frac{Z_{2b}}{Z_{2a}}$$

and

5. In order to have the arithmetical sum of the kv-a. outputs of the transformers equal to the kv-a. of the combined load the impedance angles must be equal. That is, the constants of the transformers must be so proportioned that

$$\frac{X_{2a}}{R_{2a}}=\frac{X_{2b}}{R_{2b}}$$

56. Polarity Test; Subtractive Polarity; Additive Polarity.—

Before any attempt is made to interconnect transformer windings it is necessary to know the polarities of the terminals of the respective windings. An excellent way for determining polarities is shown in Fig. 38. Two voltmeters are used. One of the meters indicates the voltage across the high-voltage winding and the other meter indicates the sum or difference of the voltages across the high- and low-voltage windings. Referring to the illustration, if the voltage from *m* to *o* is less than the voltage from *m* to *n*, then the voltages of the windings buck with reference to the mesh *mopn* and during the major portion of a cycle the terminals *m* and *o* have like polarity. This type of polarity is

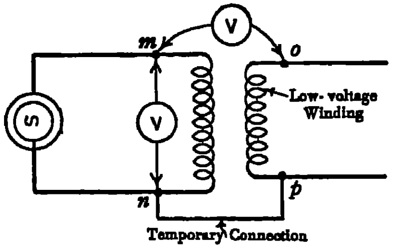


FIG. 38.—Polarity Test of a Transformer

called *subtractive polarity*. If the voltage from m to o is greater than the voltage from m to n it is evident that for the major portion of a cycle the terminals m and o have unlike polarity or what is known as *additive polarity*.

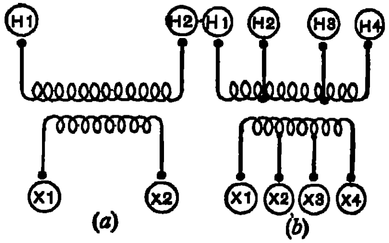


FIG. 39.—Subtractive Polarity of Transformer Windings

57. Examples of Transformer Terminal Marking.—The standard way for marking the terminals of the high-voltage winding of a transformer is by the symbols (H1), (H2), (H3), etc. Similarly the standard symbols for the terminals of the low-voltage winding are (X1), (X2), (X3), etc. (See A.I.E.E. Standardization Rules in regard to the proper way for marking transformer terminals.)

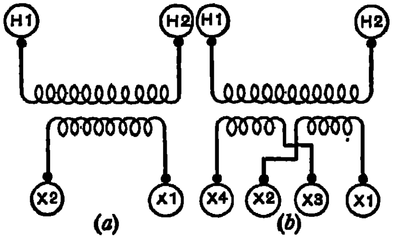


FIG. 40.—Additive Polarity of Transformer Windings

Subtractive and additive polarity are illustrated by Fig. 39 and Fig. 40 respectively.

58. Transformer Connections.—

Transformation from three-phase to three-phase using three transformers.—The high-voltage and low-voltage windings of the transformers may be connected in Y or in Δ. The connections used in any given case depend somewhat on the voltage required and on the desirability of having the neutral available. The following connections are possible.

High-voltage Winding	Low-voltage Winding	Terminal Voltage Ratio $\frac{V_H}{V_L}$
Y	Y	U
Δ	Δ	U
Y	Δ	$\sqrt{3}U$
Δ	Y	$\frac{U}{\sqrt{3}}$

In the Y–Y connection where no neutral return circuit is used a sinusoidal primary terminal voltage results in a sinusoidal secondary terminal voltage, but a non-sinusoidal Y-phase voltage. A non-sinusoidal Y-phase voltage may or may not be of importance depending on the voltage at which the transformer is operated. If the insulation of the transformer operates at a low factor of safety the wave shape of the voltage is of great importance and for this reason an explanation as to how a non-sinusoidal Y-phase voltage may be obtained is given.

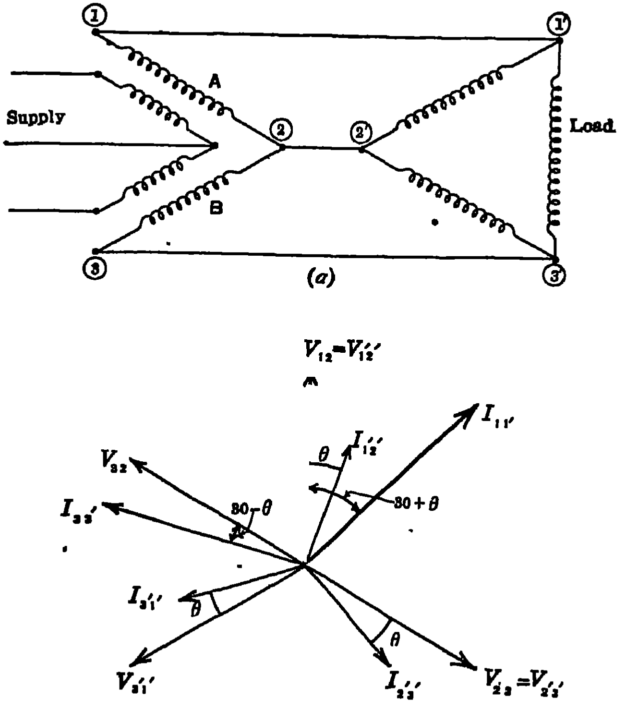
It was demonstrated in Article 42 that the exciting current required to produce a sinusoidal flux in an iron core must contain an appreciable third harmonic. No third harmonic current can be supplied by a balanced three-phase source for the simple reason that the third harmonic currents in the phases of a Y-connected load are in time phase; i.e., they must flow toward or away from the neutral at the same time. The omission of the third harmonic in the exciting current results in a flat-topped flux wave containing a third harmonic (this may be demonstrated by reversing the process in Fig. 29; i.e., assume a sine-wave current and obtain the flux wave). A flat-topped flux wave produced a peaked phase voltage wave (the maximum voltage depends on the maximum rate of change of the flux which is greater for a flat-topped wave than for a pure sine-wave representing an equal flux) whose crest value may endanger the Y-phase insulation.

If a neutral connection is provided, or if either the primary or secondary windings are connected in closed delta, a path is obtained for the circulation of the third harmonic in the exciting current. Under these conditions sine-wave voltages across the primary windings produce sine-wave voltages across the different phases and across the secondary terminals.

Open delta or V-connection.—In transforming from three-phase to three-phase it is sometimes economical to use two instead of three transformers. The windings of the transformers are connected as shown in Fig. 41*a*; the connection resembles the Δ-connection with one of the transformers left out. If the primary windings of the transformers are connected to a balanced three-phase supply, the load voltages $V_{1'2'}$, $V_{2'3'}$, and $V_{3'1'}$, are approximately equal and 120 degrees out of phase (see Fig. 41*b*).

The outputs of the transformers may be calculated when the voltages, currents, and power factors of the secondary windings are known. Thus from Fig. 41*b* the current in the secondary winding of transformer *A* in the sense 2–1 lags the voltage rise in the same sense by an angle of $30+\theta$ degrees and the power output of this transformer is

$$P_A = V_{21}I_{21} \cos (30+\theta) = VI \cos (30+\theta)$$



(*a*)

(*b*)

FIG. 41.—Open Delta Connection of Transformers

Similarly the output of transformer *B* is

$$P_B = V_{23}I_{23} \cos (30-\theta) = VI \cos (30-\theta)$$

The total power transmitted by the two transformers is

$$P = VI \cos (30+\theta) + VI \cos (30-\theta) = \sqrt{3}VI \cos \theta \qquad (11)$$

The power rating of a closed delta connection using three instead of two transformers is

$$P = 3VI \cos \theta \qquad (12)$$

where V is the load voltage, and I the current rating of each secondary winding. By comparing expressions (11) and (12) it is apparent that three transformers connected in closed delta have 1.732 times the rating of two of these transformers connected in open delta.

Two-phase to three-phase transformation (T-connection).—Sometimes it is necessary to operate two-phase apparatus on a three-phase power supply or vice versa. The interconnection may be accomplished by using two T-connected transformers. Two similar transformers having 50, 86.6, and 100 per cent winding taps are used. The transformers are interconnected as shown in Fig. 42*a*. The voltage and current relations are shown by Fig. 42*b*.

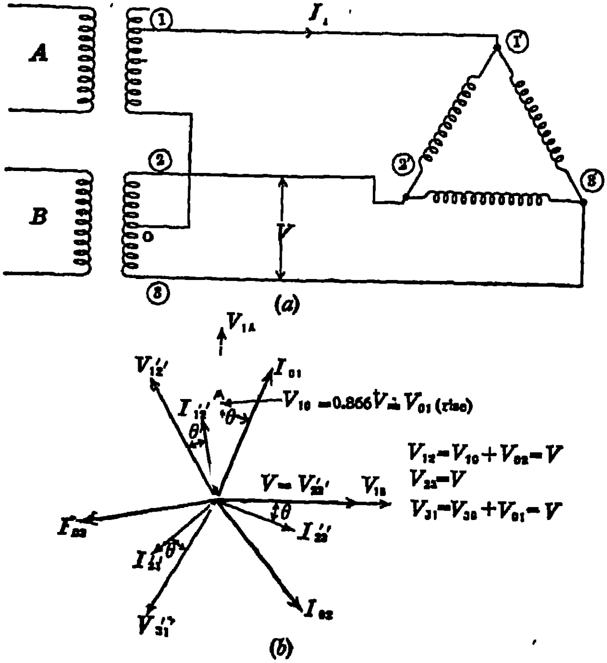


FIG. 42.—T-connected Transformers

The magnetic reactions of the secondary windings may be calculated when the currents through the windings are known. For a balanced three-phase load in which the line current is I_2 the magnetic reaction of the secondary winding of transformer A is proportional to the product $(0.866N_2)I_2$. This quantity may be represented by a vector in phase with the current I_{01} (Fig. 42*b*); the vector lags the primary voltage V_{1A} by approximately θ degrees. Similarly the magnetic reaction due to the secondary current of transformer B is proportional to the vector sum of the ampere-turns in the two halves of the secondary winding. The ampere-turns in the portions of the secondary winding 3–0 and 0–2 are each equal to $\frac{1}{2} N_2 I_2$, but they are 60 degrees out of phase due to the fact that the currents 3–0 and 0–2 are displaced by that angle. The resultant ampere-turns 3–2 are therefore equal to $[(\frac{1}{2} N_2) I_2 \cos 30]2 = 0.866 N_2 I_2$ and may be represented by a vector approximately θ degrees displaced from the primary voltage V_{1B}. It follows that the magnetic reactions due to the secondary currents in the two transformers are equal and bear the same time phase relations to the respective primary voltages and, since this is so, the loads on the two primary phases are identical.

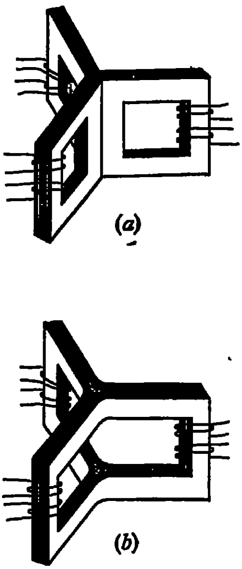
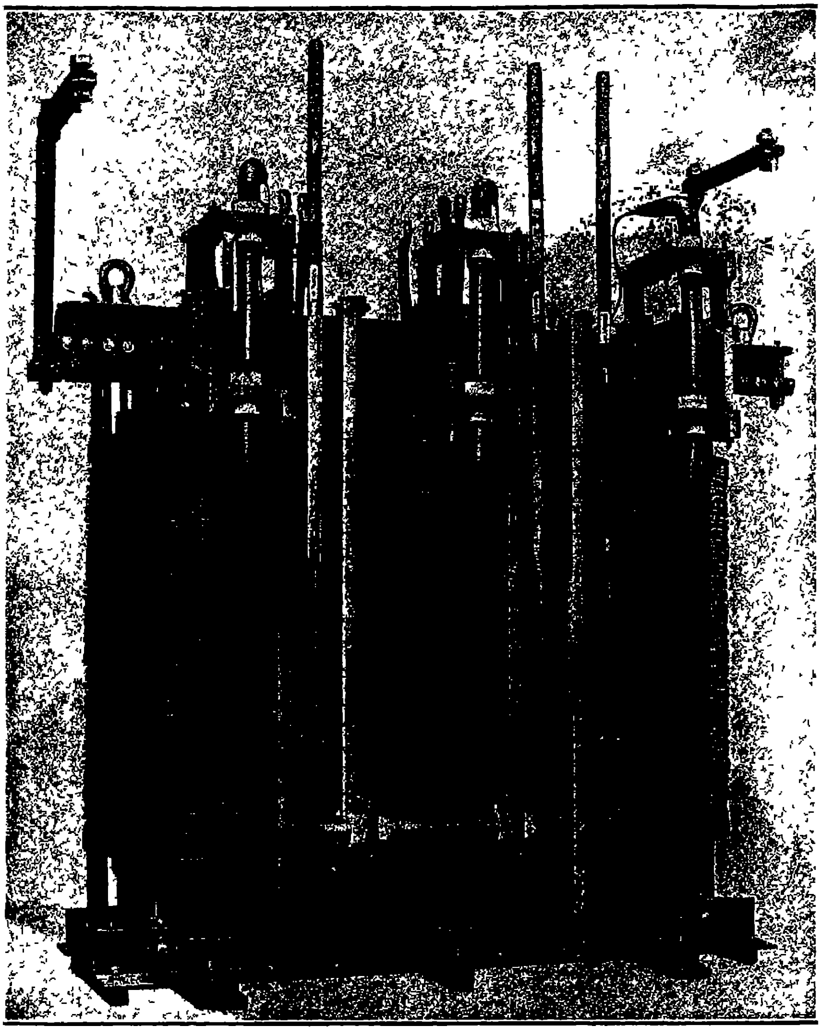


FIG. 43.—Three-phase Transformer

59. Three-phase Transformers.—Three identical single-phase transformers arranged as in Fig. 43*a* and placed in the same tank result in a three-phase transformer. When the primary windings of this transformer are connected to a balanced three-phase supply, the resultant flux through the central leg of the core is zero, and this part of the core may be entirely eliminated as shown in Fig. 43*b*. This form of construction reduces the amount of material required for a given power transformation. Fig. 44 shows the actual arrangement of the core and the windings of a three-phase core-type transformer. A three-phase transformer is cheaper than three single-phase transformers connected for equal power transformation, but a spare three-phase transformer costs considerably more than a spare single-phase transformer.

60. Tertiary Winding.—High-voltage three-phase transformers which are to be operated Y-connected are sometimes provided with a third winding. The third winding is called the tertiary winding and is delta-connected. The object of this additional winding is to provide a path for the circulation of a third harmonic current required to obtain sinusoidal core flux (see Article 42). In Article 58 it is explained that the purpose in maintaining a sinusoidal core flux is to avoid excessive crest values in the Y-phase voltages.



(By the courtesy of the General Electric Co.)

FIG. 44.—Three-phase Transformer

61. Auto-transformer.—In addition to the usual type of transformer in which the windings form two separate insulated circuits there is another type known as the auto-transformer in which the windings form a single electric circuit.

Figure 45 shows how the winding of an auto-transformer is connected. A portion of the winding serves the double purpose of primary and secondary winding. For example, if the transformer is used to lower the voltage, the turns between *m–o* constitute the primary winding and those between *n–o* the secondary winding. On the other hand, if the transformer is used to raise the voltage, the turns between *n–o* serve as the primary winding, and those between *m–o* as the secondary winding.

The ratio of transformation depends on the relative number of turns between *m–o* and *n–o*. The winding is usually provided with taps enabling a ready change in the voltage ratio. When only one ratio is required the currents in the sections of the winding *m–n* and *n–o* are different and permit these sections to be wound with wire of different sizes. The saving in copper by using an auto-transformer, instead of a two-circuit transformer is large when the ratio of transformation is small (under these conditions only a small portion of the winding carries the total load current).

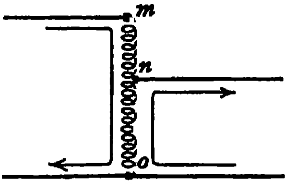


FIG. 45.—Auto-transformer

62. Voltage and Current Relations in an Auto-transformer.—

Let N_1 = turns between *m–o*.
N_2 = turns between *n–o*.
V_1 = voltage between *m–o*.
V_2 = voltage between *n–o*.

Since all the turns link the same mutual flux and since the resistance and leakage reactances of the windings are small it follows that

$$\frac{V_1}{V_2}=\frac{N_1}{N_2} \quad \text{(approximately)} \tag{13}$$

Since

$$V_1I_1 \cos \theta_1 = V_2I_2 \cos \theta_2 \quad \text{(approximately)}$$

and since

$$\cos \theta_1 = \cos \theta_2 \quad \text{(approximately)}$$

it follows that

$$V_1I_1 = V_2I_2 \quad \text{(approximately)}$$

or

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \quad \text{(approximately)} \qquad (14)$$

Note that the current in the winding *n–o* is neither I_1 nor I_2 but is equal to the vector difference of these quantities. If the ratio of transformation is small, I_1 and I_2 are nearly equal with the result that the current through the winding *n–o* is small and consequently this part of the winding may be designed for small current capacity.

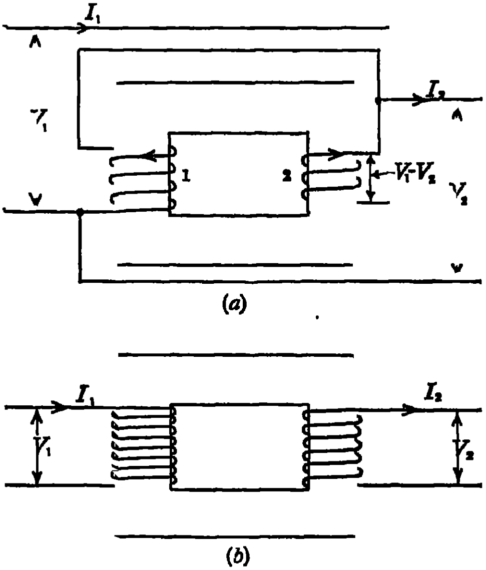


FIG. 46.—Connections of an Auto- and Two-circuit Transformer to Supply a Given Load

The winding between *m–n* must be designed to carry the total current I_1.

63. Power Rating of a Two-circuit Transformer Used as an Auto-transformer Compared with that of a Two-circuit Transformer to Supply a Given Load.—Fig. 46*a* illustrates how a two-circuit transformer may be connected to operate as an auto-transformer. In this particular scheme the connections are so made as to obtain a step-down transformer. A given load requiring V_2 volts and I_2 amperes may be supplied by this auto-transformer provided the winding No. 2 is designed for $V_1 - V_2$ volts and I_1 amperes. The rating of the transformer required for the purpose depends on the voltage and current requirements of this winding. Thus, the rating of the two-circuit transformer connected as an auto-transformer required to supply the given load may be expressed by the relation

$$P_A = (V_1 - V_2)I_1 \quad \text{volt-amperes.}$$

The rating of the two-circuit transformer required to supply the same load is

$$P_T = V_2 I_2 = V_1 I_1 \quad \text{volt amperes.}$$

From these relations it follows that the relative ratings of the two transformers may be represented by the ratio

$$\frac{\text{Rating of auto-transformer}}{\text{Rating of two-circuit transformer}} = \frac{V_1 - V_2}{V_1}.$$

The saving in transformer capacity by using an auto-transformer depends on the ratio of the primary and secondary voltages. If the secondary voltage is almost equal to the primary voltage a great saving is effected by using an auto-transformer.

64. Uses for Auto-transformers.—Auto-transformers are used where the ratio of transformation is small, and where a metallic connection between the high-voltage and low-voltage side is no serious objection. This type of transformer often takes the form of a compensator used for starting polyphase induction or polyphase synchronous motors (see Fig. 47). Auto-transformers are also used for furnishing the neutral in Edison 3-wire systems and for regulating the feeder voltage of a-c. transmission systems.

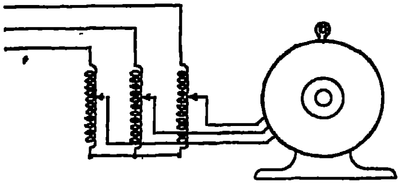
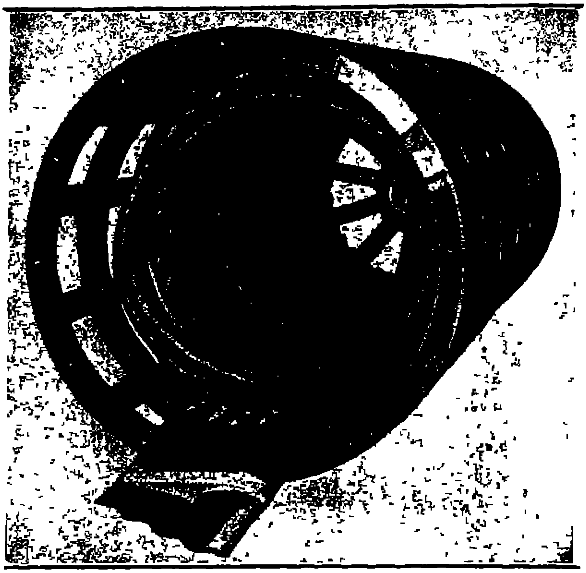


Fig. 47.—Starting Compensator for a Polyphase Motor

65. Induction Regulator.—An induction regulator is an auto-transformer in which the mutual flux of the primary and secondary windings may be altered by shifting the relative positions of these windings. The purpose of an induction regulator is to maintain the voltage at a given location at a certain definite value by raising or lowering the feeder voltage according to the requirements. The stator winding shown in Fig. 48*a* is connected in series with the feeder and the rotor winding (the high-voltage winding) shown in Fig. 48*b* is shunted across the line. Fig. 49 shows how these connections are actually made. The short-circuited winding *c* reduces the series impedance of the regulator when the rotor is located in the no-boost or buck position.

The rotors of induction regulators are usually turned by small induction motors located on the top of the regulators. The action of these motors may be controlled by a system of relays. In other words automatic operation is possible and is usually used.

66. Constant Current Transformer.—A constant current transformer is a transformer in which the current in the windings is nearly constant throughout a wide range of load impedances. The transformer is constructed as shown in Fig. 50. The secondary coil is free to move toward or away from the primary coil and the major portion of its weight is counterbalanced by the weight W.



(By the courtesy of the General Electric Co.)

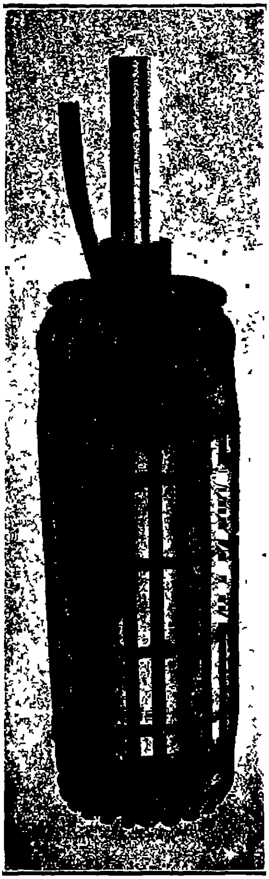
FIG. 48a.—Stator Winding of a Polyphase Induction Regulator

In actual operation the leakage magnetic fields produce a force of repulsion between the two coils and the secondary coil is lifted to a position where the resultant force acting on it is zero. One of the limiting positions of the secondary coil (coils close together) is obtained when the load has a relatively high impedance and the other limiting position (coils far apart) is obtained when the load has a low impedance. In normal operation the current in the windings is determined by the magnitude of the counter weight. If the current changes slightly from its normal value the magnetic repulsion between the coils changes, the secondary coil moves to a new position where the flux linking the winding is different. The action is such that a change in current produces an opposite change in the e.m.f. which produces the current with the result that within certain limits of load impedances the current remains nearly constant.

The chief use for constant current transformers is for furnishing power to series street-lighting systems. If in Fig. 50 an additional lamp is connected in the circuit, the current momentarily decreases, the windings move closer together, the secondary e.m.f. becomes higher, and the secondary current tends to assume its former value. Similarly a reduction in the number of lamps in the circuit results in a lower e.m.f. and approximately the same current.

67. Instrument Transformers.—Measuring instruments are usually designed to give full range deflection at 150–300 volts and at 5–10 amperes. In order to make measurements on high-voltage circuits it is necessary to use transformers whose ratios of transformation and phase angles are definitely known. The transformers and measuring instruments are connected as shown in Fig. 51*a*.

The true voltage, current, and power in the circuit can only be obtained by multiplying the meter indications by certain factors which depend on the ratios and phase angles of the instrument transformers.



(By the courtesy of the General Electric Co.)

Fig. 48*b*.—Rotor of a Polyphase Induction Regulator

The primary winding of a current transformer is connected in series with the circuit in which the current and power are to be determined and its secondary winding is connected in series with the ammeter and current coil of the wattmeter. The ratio of the primary current I_1 to the secondary current I_2 is called the ratio of the current transformer and the angle between the current I_1 and I_{21} (secondary current referred to the primary winding) is called the phase angle of the transformer. The exciting current $(\dot{I}_1-\dot{I}_{21})$ of a current transformer depends on the secondary load current and the impedances of the ammeter and wattmeter connected to its secondary winding. For this reason the ratio and the phase angle of a given transformer are not constant and calibration curves are required for these quantities.

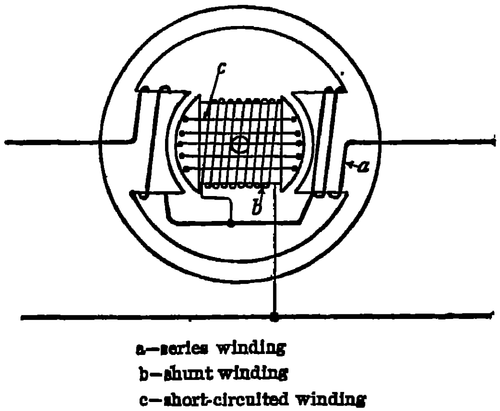


FIG. 49.—Connections of the Windings of an Induction Regulator

The primary winding of a potential transformer is connected across the circuit of which the voltage is to be measured and the secondary winding is connected across the voltmeter and potential circuit of the wattmeter. The ratio of the primary voltage to the secondary voltage is called the ratio of the transformer and the phase angle between the primary voltage and the secondary voltage referred to the primary winding is called the phase angle of the transformer. On account of the imperfections in the transformer neither the ratio nor the phase angle are constant and for this reason calibration curves are required.

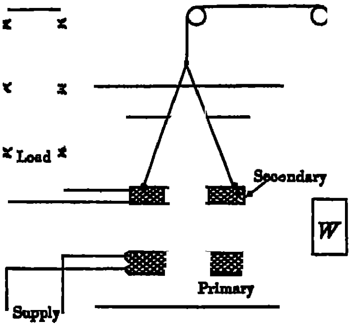


FIG. 50.—Constant Current Transformer

Assuming that the measuring instruments are accurate; the true current, voltage, and power may be obtained as follows: the true current in the circuit

$$I_1=I_2U_C$$

where I_2 is the ammeter indication and U_C is the ratio of the current transformer.

The true voltage across the circuit

$$V_1 = V_2 U_P$$

where V_2 is the voltmeter indication and U_P is the ratio of the potential transformer.

The true power in the circuit

$$V_1 I_1 \cos \theta_1 = (V_2 I_2 \cos \theta') U_C U_P \left(\frac{\cos \theta_1}{\cos \theta'}\right)$$

where $V_2 I_2 \cos \theta'$ is the wattmeter indication, U_C and U_P are the ratios of the transformers, and $\left(\frac{\cos \theta_1}{\cos \theta'}\right)$ is a correction factor

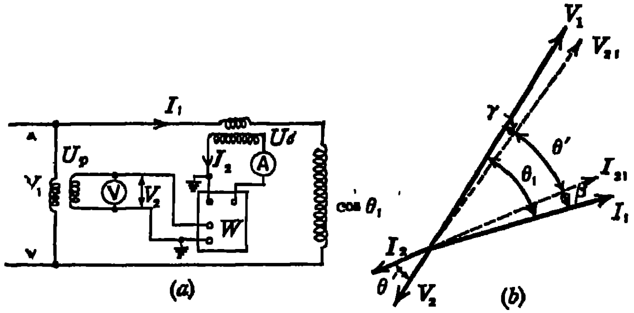


FIG. 51.—Connections of Instrument Transformers and Vector Diagram

which takes into account the phase angle errors in the transformers. The relation between θ_1 and θ' may be expressed as follows

$$\theta_1 = \theta' + \beta + \gamma$$

where θ' is the phase angle between V_2 and I_2; β is the phase angle of the current transformer; and γ is the phase angle of the potential transformer.

A vector diagram showing the voltage and current relations is given in Fig. 51*b*. Note that the voltage V_1 is assumed to lead the voltage V_{21} and the current I_{21} is assumed to lead the current I_1. This is usually the case in transformers.

PROBLEMS

1. An impedance having an equivalent resistance of 10 ohms and an equivalent inductive reactance of 3.2 ohms is connected in parallel with another impedance having 8.1 ohms resistance and an adjustable reactance. The vector sum of the currents through the impedances is 200 amperes. (*a*) How must the currents be proportioned to obtain the minimum power loss in the impedances? (*b*) Prove the relations used in solving part *a*.

2. Two 140,000 to 5000-volt, 60-cycle transformers are operated in parallel on both the primary and secondary sides. The following things are known about the transformers:

Rating	2000 Kv-a.	5000 Kv-a.
Eff. resistance of high-voltage winding	20.00 ohms	12.20 ohms (at op. temp.)
Eff. resistance of low-voltage winding	0.07 ohm	0 023 ohm (at op. temp.)
Iron loss	23,900 watts	41,350 watts
No-load current	7.2 per cent	7.0 per cent
Reactance drop	8.80 per cent	8.2 per cent
Turn ratio	28	28

The load voltage is maintained constant at 5000 volts.

Calculate: (*a*) the current circulating through the secondary windings at no-load; (*b*) the primary voltage, current, and power factor of each transformer when the combined load is 7000 kv-a., 90 per cent lagging power factor; and (*c*) the nature of the impedance which must be connected in the secondary leads of one of the transformers so as to make them share the load properly.

3. Two identical transformers are connected in open-delta on the primary and the secondary sides. The secondary windings are connected to a balanced three-phase load requiring 628 kv-a. at 550 volts. The power factor of the load is 85 per cent lagging. (*a*) Calculate the kv-a. ratings of the transformers required to supply this load. (*b*) Calculate the power factor of each secondary winding. (*c*) How many answers are possible in part (*b*)? (*d*) Draw a complete voltage-current vector diagram showing the voltage and current relations in part (*b*).

4. Two identical step-down transformers are used for the purpose of transforming from two-phase to three-phase. Each secondary winding is provided with 86.6, and 50 per cent winding taps. The secondary windings are T-connected and supply a 750-kv-a., balanced three-phase, 90 per cent lagging power factor load. The load voltage, measured at the terminals of the transformers is 440 volts. The turn ratio is 5. (*a*) Calculate: (1) the output of each secondary winding; (2) the current and power factor of each primary winding. (Assume that the no-load current of the transformers is negligible.) (*c*) Construct a complete voltage-current vector diagram showing all the possible voltage and current relations.

5. A load requiring 800 kv-a. at 550 volts must be supplied from a 440-volt source. Determine the relative capacities of an auto-transformer and a two-

circuit transformer connected as an auto-transformer required to supply this load.

6. An ammeter, voltmeter, and wattmeter are connected for measuring the power and the power factor of a single-phase load. The voltage and the current are stepped down by instrument transformers (see Fig. 51). The following things are known:

Wattmeter indication 67 watts (the wattmeter is accurate when no instrument transformers are used to supply it);
Ammeter indication, 2.1 amperes (accurate);
Voltmeter indication, 115 volts (accurate);
Marked ratio of voltage transformer 10;
Marked ratio of current transformer 8;
Ratio and phase-angle of voltage transformer obtained from calibration curves; ratio 9.9, phase-angle 0.16 degree (referred secondary voltage lags primary voltage);
Ratio and phase-angle of current transformer obtained from calibration curves; ratio 8.10, phase-angle 1.2 degrees (referred secondary current leads the primary current).

(*a*) Calculate: (1) the line voltage and the line current; (2) the apparent power and power factor of the circuit (no correction made for ratios and phase-angles); (3) the true power and power factor of the circuit; and (4) the per cent error in the power indication of the wattmeter due solely to the phase-angle errors of the instrument transformers.

(*b*) Given: total power transmitted 20 kilowatts; line voltage 1100; and power factor unity. Calculate: (1) ammeter indication; (2) voltmeter indication; (3) wattmeter indication; and (4) the percentage errors in (1), (2), and (3). [(Assume the true ratios and phase-angles the same as in part (*a*).]

CHAPTER V

ALTERNATING-CURRENT GENERATOR

68. A-c. Generator or Alternator.—Fig. 52 represents an a-c. generator in one of its simplest forms. The essential parts are a magnet and an electric circuit. In order to obtain generator action a relative motion must take place between these parts.

The direction of the magnetic field through the electric circuit *ABCD* is up or down, depending on the position of the revolving magnet. The repeated reversal of the field through the circuit induces an alternating e.m.f. in it. The time wave shape of the e.m.f. depends on the space wave shape of the magnetic field. The frequency of the e.m.f. depends on the number of poles and on the relative speed of the field and armature. The r.m.s. value of the e.m.f. depends on the magnitude of the field, the wave shape of the field, the kind of armature winding, and the relative speed of the field and armature.

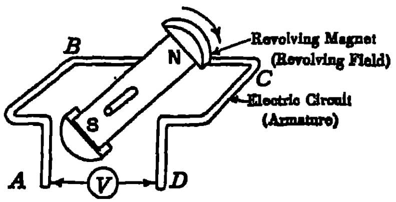


FIG. 52.—Simplified Alternating-current Generator

In Fig. 52 the armature is shown as stationary and the field as revolving. This is the usual form of construction used on large machines. Small alternators used in machinery laboratories are usually equipped with armatures that may be rotated in stationary fields.

69. Revolving Armature Type of Alternator.—In this type of construction the total power generated in the armature must be collected by brushes bearing on collector-rings. Since sliding contacts are unsatisfactory when the power is large or when the voltage is high this type of construction is limited to small machines only. The revolving armature type of alternator is similar to a d-c. generator, the main difference being that the commutator is replaced by collector-rings. The number of collector-rings depends on the number of phases; a single-phase machine requires two rings and a three-phase machine three (see Figs. 53*a* and *b*).

The field winding is excited by a separate d-c. generator or by a storage battery. Alternating-current excitation cannot be used on account of the difficulty involved in producing a large alternating-flux.

70. Revolving Field Type of Alternator.—All large alternators are equipped with revolving fields, the reason being that only a small power (the field exciting power) must be supplied through collector-rings.

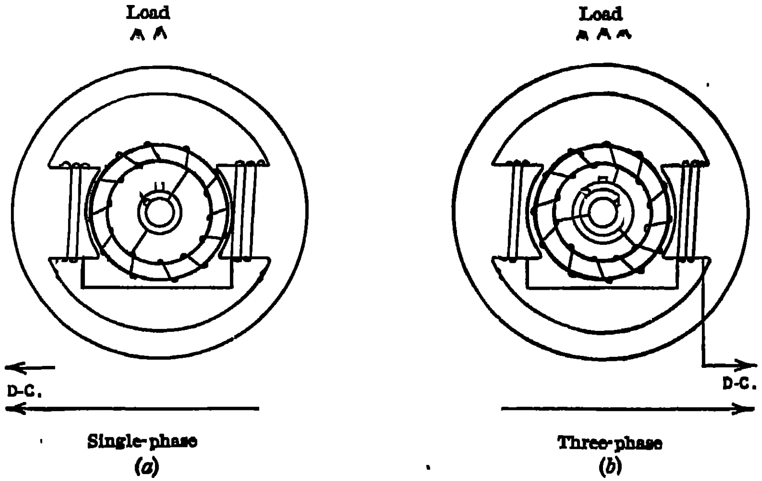
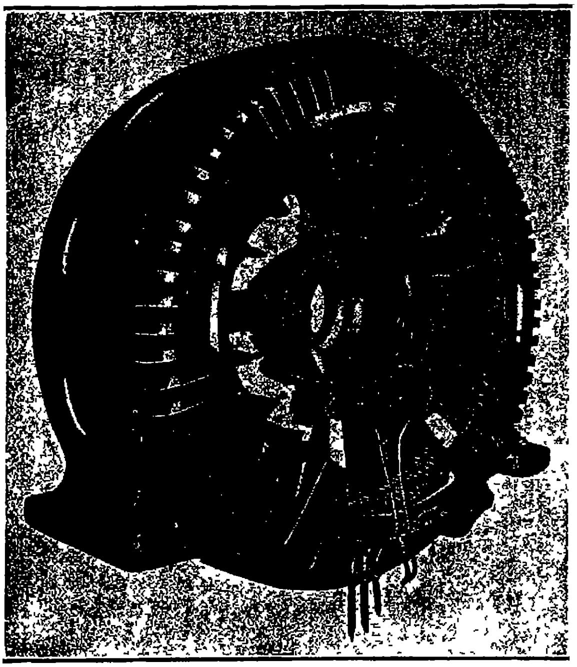


FIG. 53.—Revolving Armature Type of Alternator

The revolving fields may be equipped with many or few poles, depending on the speed and the frequency desired. Alternators driven by reciprocating engines or by water wheels operate at relatively slow speeds and require many poles. Turbo-alternators operate at high speeds and usually have two or four poles.

71. Slow-speed Alternators.—Fig. 54 represents a partly assembled engine-driven alternator. The stator (the armature) is made of laminated iron which is supported by an iron frame work. The armature winding is embedded in the stator slots and is distributed uniformly around the inside periphery of the stator.

The rotor (the revolving field) has laminated iron poles which are securely bolted to an iron spider. The field coils are wound with strap copper and have the appearance of the coil shown in Fig. 55. This type of field winding is used, because it is readily cooled (one edge of the copper strap is exposed to the cooling air) and because it is mechanically very strong.



(By the courtesy of the Westinghouse Co.)

FIG. 54.—Engine-type of Alternator

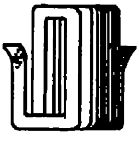
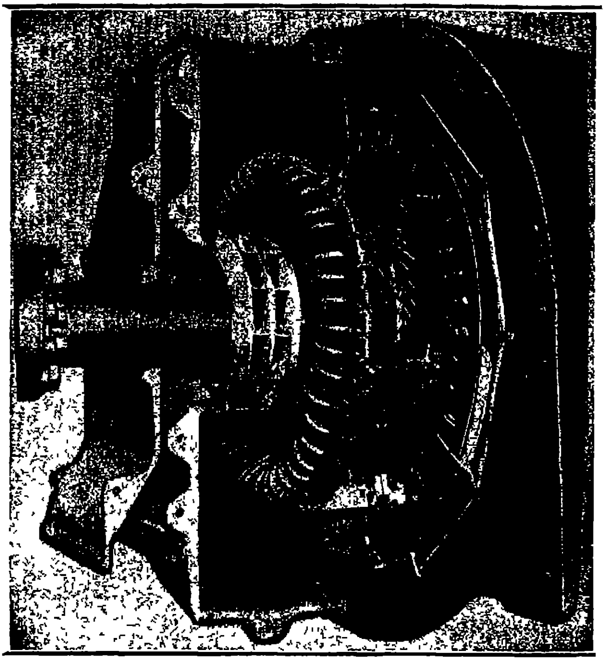


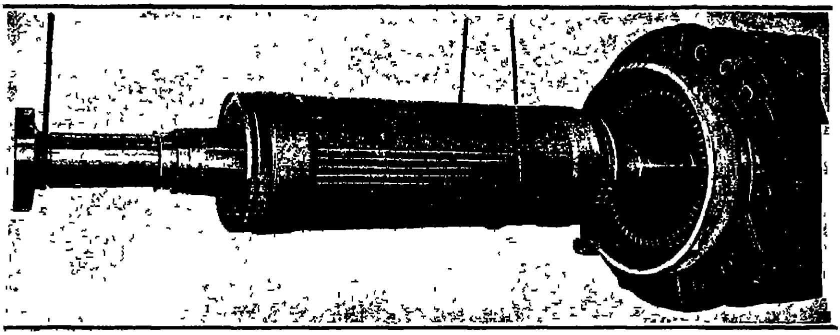
FIG. 55. — Field Coil Wound with Copper Strap

72. High-speed Alternators.—Fig. 56 represents a partly assembled high-speed alternator known as a turbo-generator. All steam turbines operate at relatively high speeds and for this reason the generator which is driven by the turbine must be specially constructed to operate at these speeds. For example, the rotor is subjected to centrifugal forces which are very great and for this reason its diameter must be kept small and its field winding must be thoroughly secured.



(By the courtesy of the Westinghouse Co.)

FIG. 56.—Turbo-generator Showing Armature Winding, Rotor and Ventilating Passages



(By the courtesy of the General Electric Co.)

FIG. 57.—Rotor of a Turbo-generator

A typical rotor is shown in Fig. 57; its length is considerably greater than its diameter. Non-salient magnet poles are used and the field winding is embedded in several slots (Fig. 58) so as to obtain an approximately sinusoidal field form. The field is wound with strap copper, is mica insulated, and is secured in the rotor slots by metallic wedges. The end connections of the field winding are secured by metallic rings having a low magnetic permeability. In short the rotor is constructed for mechanical strength, sine-wave field form, a small amount of air friction, and for minimum noise in operation.

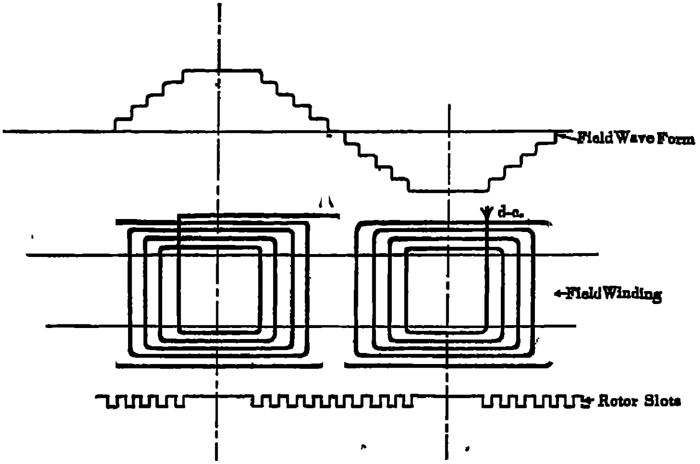


FIG. 58.—Field Winding and Field Wave-form of a Non-salient Pole Turbo-generator

The armature is wound with diamond-shaped coils. Each coil has one or more turns and each turn usually has several conductors in parallel. The kind and quantity of insulation on the turns and coils depends on the voltage and on the operating temperature. Mica is one of the best materials available for insulating armature and field windings.

73. Inductor Alternators (Figs. 59*a* and 59*b*).—Inductor alternators are revolving field alternators in which the exciting winding is stationary and no sliding contacts are required in either the field or the armature circuits. The chief difficulties with this type of construction are that two armature windings are required and that the full benefit of the flux in generating e.m.f.'s is not obtained.

The flux through armature winding A pulsates between the values of 0 and ϕ_N and that through winding B between the values 0 and ϕ_S. A pulsating flux induces only one-half of the voltage induced by an alternating-flux of the same maximum value. For this reason inductor alternators require more armature and field material than the usual type of alternator and are used in special applications only.

74. Alternator Armature Windings.—In order to distribute the heating over as large a surface as possible, alternator armature windings are always located in many armature slots. The slots are uniformly spaced over the entire surface of the armature. Each phase of a polyphase generator occupies several slots per pole per phase and all phases combined occupy the total available winding space. In single-phase machines some of the armature slots are left vacant for the reason that it is uneconomical to make the width of one phase band 180 electrical space degrees (see Article 79).

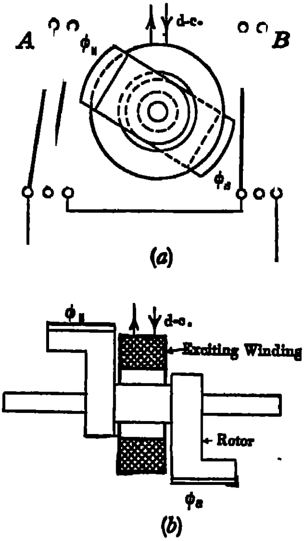


Fig. 59.—Inductor Alternator

The power rating of an alternator depends on the current and voltage rating of its armature winding. The current rating of the armature is determined by the heat developed in the generator which in turn depends on the actual current delivered and on the power factor of the load. The voltage rating depends on how the machine is wound.

Alternators are wound so as to obtain a nearly sinusoidal voltage wave and a high power rating for the quantity of material used. A given armature winding connected for polyphase operation has a higher power rating than the same winding connected for single-phase operation. For example a mesh-wound single-phase alternator like that in Fig. 53*a* having a rating of 3 kv-a. at 100 volts will have a rating of $\sqrt{3}(100\times0.866)\left(\frac{\sqrt{3}\;3000}{100\times2}\right)\left(\frac{1}{1000}\right)$ $=3.9$ kv-a. at 86.6 volts when connected three-phase as shown in Fig. 53*b*.

A typical armature winding for a 4-pole, Y-connected, three-phase alternator is shown in Fig. 60. In this particular winding a given coil spans $\frac{2}{3}\pi$ electrical space radians and the coil pitch is said to be $\frac{2}{3}\pi$. The number of coil groups in a given phase is equal to the number of magnet poles and for this reason the winding is said to be a full-coil winding.

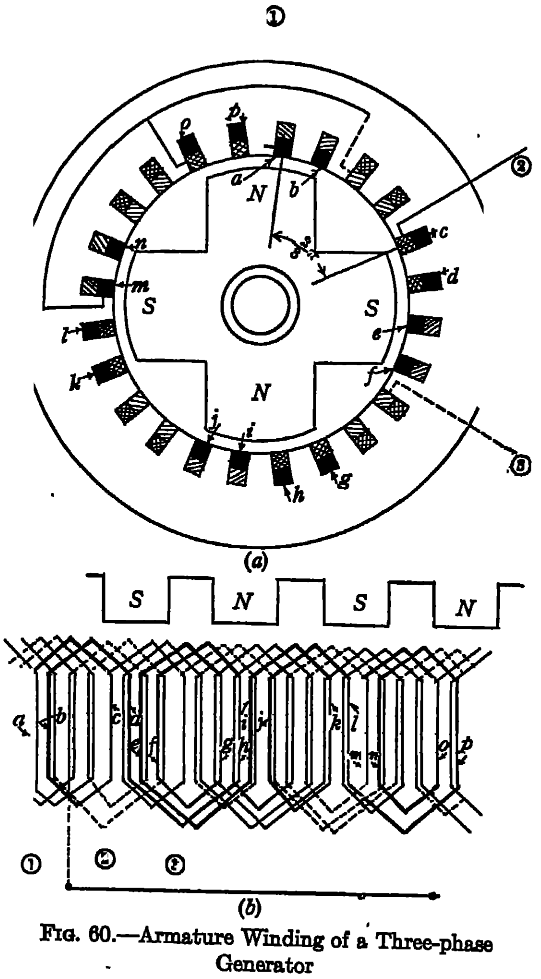


FIG. 60.—Armature Winding of a Three-phase Generator

75. Methods Used for Cooling Alternators.—The capacity of an a-c. generator is limited by the maximum safe operating temperature. All a-c. generators are air-cooled. Small generators of the open type of construction are readily cooled by the fanning action of the rotor. High-capacity generators are difficult to cool; the air must be forced through the generator by a special fan.

The quantity of cooling air required to keep a generator cool may be calculated on the basis that a 1 kilowatt loss acting for 1 minute raises the temperature of 90 cubic feet of air 20° C. On this basis (a 20° rise in temperature) a generator delivering 20,000 kilowatts at 96.7 per cent efficiency requires 59,400 cubic feet of air per minute.

High-capacity generators usually operate at relatively high temperatures. It is not uncommon that the insulation of the windings is subjected to temperatures as high as 150° C. This is one of the main reasons why mica insulation is used. The operating temperatures of large generators are determined by thermocouples located in the generator at places where the temperature is likely to be the highest.

There are three schemes used for cooling or ventilating generators, namely: radial ventilation, circumferential ventilation, and axial ventilation. The three methods are illustrated by Figs. 61*a*, 61*b*, and 61*c*. In any particular machine a combination of the methods is usually used. In radial and circumferential ventilation the cooling air is blown along the surface of the laminations while in axial ventilation the cooling air is blown across the laminations. Both radial and circumferential ventilation require that the heat generated in the stator iron be conducted across the laminations to the air ducts. The heat conductivity across a stack of laminations is only about 10 per cent of the heat conductivity along the laminations. From this standpoint the axial method of ventilation which requires that the heat be conducted along the laminations to air passages is preferred.

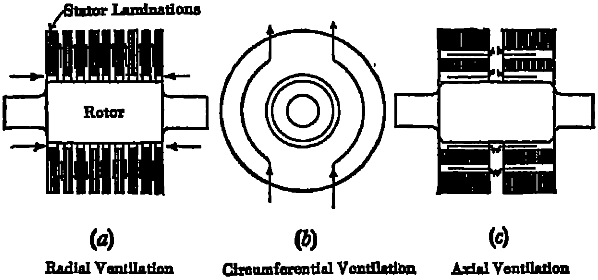


Fig. 61.—Methods for Cooling Generators

Foreign matter suspended in the cooling air is likely to clog the air passages and ruin the insulation of the windings. For this reason the air is sometimes washed by passing it through sprays of water. Washing not only cleans the air but also lowers its temperature and increases its humidity. All of these factors result in a higher safe power rating for the generator.

CHAPTER VI

THEORY OF OPERATION OF ALTERNATORS

76. Frequency of the Voltage Generated by an Alternator.—The flux through a given armature coil reverses every time the coil passes over a field pole. Since two complete reversals of the flux are required to generate a complete cycle in the e.m.f. wave it follows that the frequency of the e.m.f. generated by an alternator having p poles and making n r.p.s. is

$$f=\frac{p}{2}n \text{ cycles per sec.} \qquad (1)$$

The standard frequencies now employed in this country are 25 and 60 cycles per sec. Twenty-five cycle current is unsatisfactory for lighting, but both frequencies are satisfactory for power transmission and power utilization. Most of the recent power installations favor the 60-cycle frequency.

77. Voltage Generated in an Armature Coil.—In order to calculate the voltage generated in an armature coil it is necessary to know the number of turns in the coil, the coil pitch, the flux per pole, the space distribution of the flux, and the speed of the rotor.

Let N_c = number of turns in an armature coil;
a = coil pitch measured in electrical space radians (the distance between centers of adjacent poles is π electrical space radians);
l = effective length of armature coil (the part which cuts the flux);
and B_1, B_3, B_5 = the maximum flux densities due to the fundamental, third harmonic, and fifth harmonic in the space distribution of the field in the air-gap.

The armature in Fig. 62 is assumed to rotate, to the right and at a speed of $2\pi\left(\frac{p}{2}n\right)$ electrical space radians per second. Since the frequency of the e.m.f. generated is equal to $\frac{p}{2}n$, it follows, that at a given time the speed of the armature in electrical space radians per second is numerically equal to $2\pi f$ electrical time radians of the induced e.m.f. per second.

The flux density in the air-gap at a point P located at a distance x from the reference axis YY, drawn through the center of a north pole, is

$$\mathcal{B}_x = B_1 \cos(x \pm \theta_1) + B_3 \cos 3(x \pm \theta_3) + B_5 \cos 5(x \pm \theta_5) \qquad (2)$$

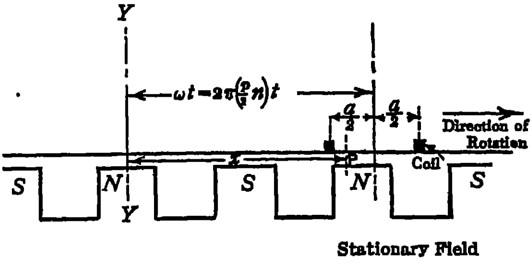


FIG. 62.—Section Through the Armature and Field of a Generator

An assumption is made that the wave form of the field contains no even harmonics. The basis for this assumption is that the air-gap under each field pole is the same and that the machine is symmetrically constructed throughout.

The total flux linking the armature coil for any position of the armature is

$$\phi_a = l \int_{\omega t - \frac{a}{2}}^{\omega t + \frac{a}{2}} \mathcal{B}_x dx \qquad (3)$$

Since the total flux linking the armature coil due to the nth harmonic is

$$\phi_n = l \int_{\omega t - \frac{a}{2}}^{\omega t + \frac{a}{2}} B_n \cos n(x \pm \theta_n) dx = \frac{2}{n} l B_n \sin \frac{na}{2} \cos n(\omega t \pm \theta_n) \qquad (4)$$

It follows that the flux linking the coil due to all the harmonics is

$$\phi_a = 2l\left[B_1 \sin\frac{a}{2}\cos(\omega t \pm \theta_1) + \frac{B_3}{3}\sin\frac{3a}{2}\cos 3(\omega t \pm \theta_3) + \frac{B_5}{5}\sin\frac{5a}{2}\cos 5(\omega t \pm \theta_5) + \text{etc.}\right] \qquad (5)$$

The voltage generated in the armature coil due to the nth harmonic in the space distribution of the flux is

$$e_{nc} = -N_c\frac{d\phi_n}{dt}10^{-8} = 2\omega l N_c B_n 10^{-8}\sin\frac{na}{2}\sin n(\omega t \pm \theta_n) \qquad (6)$$

and the instantaneous e.m.f. due to all the harmonics is

$$e_c = 2\omega l N_c 10^{-8}\Big[B_1 \sin\frac{a}{2}\sin(\omega t \pm \theta_1) + B_3 \sin\frac{3a}{2}\sin 3(\omega t \pm \theta_3) + B_5 \sin\frac{5a}{2}\sin 5(\omega t \pm \theta_5) + \text{etc.}\Big] \qquad (7)$$

The product $2lB_1$ is equal to the quantity $l\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} B_1 \cos x dx$ and represents the maximum flux linking a full-pitch armature coil due to the fundamental wave in the space field form. This product may be called the fundamental flux per pole; i.e., $2lB_1 = \phi_{1p}$, Similarly $2lB_3 = \phi_{3p}$ etc.

The e.m.f. generated due to the fundamental in the field form is

$$e_{1c} = \omega N_c \phi_{1p} 10^{-8}\sin\frac{a}{2}\sin(\omega t \pm \theta_1) \qquad (8)$$

and the r.m.s. value of this e.m.f. is

$$E_{1c} = \frac{\omega \phi_{1p} N_c 10^{-8}}{\sqrt{2}}\sin\frac{a}{2} = 4.44 N_c f \phi_{1p} 10^{-8}\sin\frac{a}{2} \qquad (9)$$

If the coil sides are π electrical space radians apart, $\sin\frac{a}{2} = 1$, and the r.m.s. value of the e.m.f. becomes

$$E_{1c} = 4.44 N_c f \phi_{1p} 10^{-8} \qquad (10)$$

The student should compare this equation with that derived for the induced e.m.f. of a transformer.

78. Elimination of the Harmonics in the E.M.F. Wave of an Alternator.—A study of equation (6) shows that any coil pitch which makes the product $\frac{na}{2}$ equal to $k\pi$ where k is an integer, results in the elimination of the nth harmonic in the e.m.f. Thus by making the coil pitch $\frac{2}{3}\pi$ or $\frac{4}{3}\pi$ the third harmonic may be eliminated. Similarly, by making the coil pitch $\frac{2}{5}\pi$, $\frac{4}{5}\pi$, $\frac{6}{5}\pi$, or

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$\frac{4}{5}\pi$ the fifth harmonic may be eliminated. The coil pitch should be so chosen that the coils span approximately π electrical space radians. Coils having a very short or a very long pitch are unsatisfactory because by their use the fundamental in the e.m.f. wave is greatly reduced.

79. E.M.F. Generated in a Coil Group.—A coil group consists of several coils located in adjacent slots and connected in series. The e.m.f. generated in a coil group depends on the e.m.f. generated in a coil, the number of coils in the group, and on a factor which is determined by the way the coils are distributed in the slots.

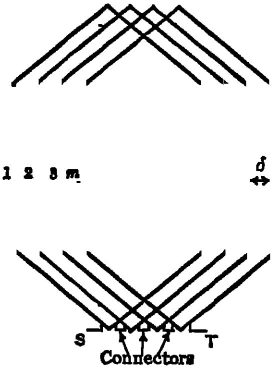


FIG. 63.—Coil Group

Figure 63 represents a coil group having m coils connected in series. Corresponding coil sides of adjacent coils are placed in adjacent slots. The distance between adjacent slots is

$$\delta = \frac{p}{2}\frac{360}{S} \qquad (11)$$

electrical space degrees. Where S represents the number of slots and p the number of poles.

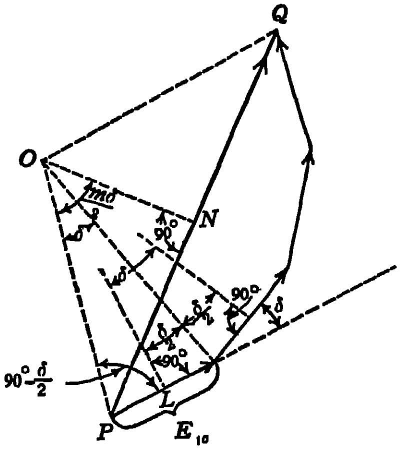


FIG. 64.—Vector Diagram Showing How to Determine the Voltage Generated in a Coil Group

If the space distribution of the air-gap flux is a sine-wave, the e.m.f.'s in coils 1 and 2, 2 and 3, etc., are δ time degrees out of phase and the e.m.f. across the terminals ST of the coil group is equal to the vector sum of the e.m.f.'s in the coils. Thus, if in Fig. 64, the vector E_{1c} represents the r.m.s. value of the e.m.f. in one coil then the total e.m.f. across the coil group is equal to the vector PQ. This e.m.f. is less than the quantity mE_{1c} which represents the e.m.f. when all the coils belonging to the group are concentrated in the same slots. A factor by which the quantity mE_{1c} must be multiplied in order to obtain the true terminal e.m.f. of a coil group is obtained as follows: Referring to Fig. 64 it may be shown that

$$PQ = E_{1c}\frac{\sin\frac{m\delta}{2}}{\sin\frac{\delta}{2}} \tag{12}$$

and that the factor

$$K_2 = \frac{PQ}{mE_{1c}} = \frac{\sin\frac{m\delta}{2}}{m\sin\frac{\delta}{2}} \tag{13}$$

represents the factor desired.

The r.m.s. value of the e.m.f. generated in a coil group due to the fundamental in the field form now becomes

$$E_{1cg} = 4.44N_cf\phi_{1p}(m)\sin\frac{a}{2}\left(\frac{\sin\frac{m\delta}{2}}{m\sin\frac{\delta}{2}}\right)10^{-8} \tag{14}$$

80. The Phase E.M.F. of an Alternator.—An alternator is usually wound so that the e.m.f.'s of the respective coil groups are equal. The phase e.m.f. is therefore equal to the e.m.f. per coil group times the number of coil groups per phase (the e.m.f.'s in the coil groups belonging to a given phase are usually in time phase).

Let N = total number of turns per phase
$= N_c \times (m) \times$ (coil groups per phase)

$$K_1 = \sin\frac{a}{2}$$

$$K_2 = \frac{\sin\frac{m\delta}{2}}{m\sin\frac{\delta}{2}}$$

and E_1 = the phase e.m.f. due to the fundamental in the space distribution of the field flux.

Substituting the proper values in equation (14) we obtain for the phase voltage

$$E_1 = 4.44Nf\phi_{1p}K_1K_2 10^{-8} \tag{15}$$

To determine the phase e.m.f. due to the nth harmonic in the space distribution of the flux it is necessary to modify equation (15) as follows:

$$E_n = 4.44 N f \phi_{np} \sin\left(\frac{na}{2}\right) \frac{\sin \frac{m(n\delta)}{2}}{m \sin\left(\frac{n\delta}{2}\right)} 10^{-8} \qquad (16)$$

Note that in this equation f is the frequency of the e.m.f. due to the fundamental in the space distribution of the flux and not the frequency of the harmonic under consideration [see equation (7)].

81. The Phase Terminal Voltage of an Alternator.—The phase terminal voltage of an alternator for any given load depends on the phase e.m.f. due to the air-gap flux and on the phase voltage drops due to the resistance and leakage reactance of the winding. Phase terminal voltage rise may be expressed by the relation

$$\dot{V}_1 = \dot{E}''_1 - (r_1 + jx_1)\dot{I}_1 \qquad (17)$$

where

E''_1 = phase e.m.f. due to the resultant field in the air-gap. (The e.m.f. is taken positive in the same sense in which the armature current is taken as positive.)

r_1I_1 = armature phase effective resistance drop.

x_1I_1 = armature phase leakage reactance drop.

The right-hand member of equation (17) may be modified so as to obtain the relations

$$\dot{V}_1 = \dot{E}'_1 - (r_1 + jX_1)I_1 \qquad (18)$$

or

$$\dot{V}_1 = \dot{E}'''_1 - r_1\dot{I}_1 \qquad (19)$$

where

E'_1 = armature phase e.m.f. at no-load (no armature current). This e.m.f. is often called the excitation e.m.f.

X_1I_1 = voltage drop per phase due to an e.m.f. which is set up by the flux produced by the armature current alone (no field current). This drop may be resolved into two components, one due to the leakage flux (the x_1I_1 drop) and the other the armature reaction voltage drop V_{ar} due to the armature reaction field produced by the armature load current.

E'''_1 = armature phase e.m.f. due to all the flux linking an armature phase.

82. Voltage-Current Vector Diagram for an Alternator. Fig. 65 is a vector diagram showing the voltage current relations given by equations (17), (18,) and (19).

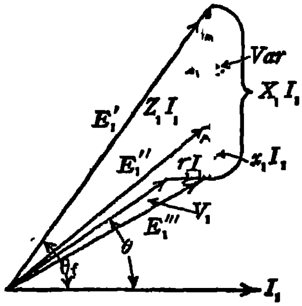


Fig. 65.—Vector Diagram for a Phase of an Alternator

From the theory of the transformer in Article 36 it follows that the drop $x_1 I_1$ due to the armature leakage flux is 90 time degrees ahead of the armature current, but it does not follow that the voltage drop V_{ar} due to the armature reaction field bears a similar phase relation to the armature current. In order to determine the armature reaction voltage and its phase relations it is necessary to determine the nature of the magnetic field produced by the armature currents acting alone.

83. Magnetic Field Produced by a Polyphase Current in a Polyphase Armature Winding (see Article 139).—The discussion which follows applies to the three-phase armature winding represented by Fig. 66. The machine is assumed to have non-salient field poles and its armature is so wound that a current set up in any one phase produces a sine-wave field form in the air-gap. A single coil shown in the figure represents, in reality, a coil group.

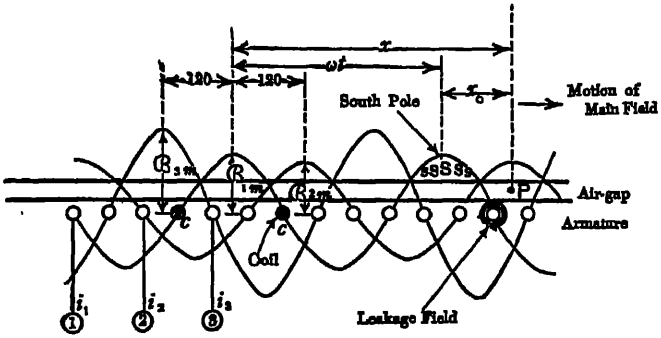


Fig. 66.—Diagram Showing Three-phase Armature Winding and the Component Magnetic Fields Produced when Positive Currents i_1 i_2, and i_3, are Established in the Three Phases

A steady current i_1 entering phase 1 through terminal (1) produces a sine-wave field form in the air-gap whose maximum density is $\mathcal{B}_{1m}$. A steady current of value i_2 entering phase 2 through terminal (2) produces a sine-wave field whose maximum density $\mathcal{B}_{2m}$ is displaced 120 electrical space degrees from the maximum density $\mathcal{B}_{1m}$. Similarly a steady current i_3 entering phase 3 through terminal (3) produces a sine-wave field whose maximum density $\mathcal{B}_{3m}$ is displaced 120 electrical space degrees from $\mathcal{B}_{2m}$.

The flux density at any point P located x electrical space degrees from the center of coil c is

$$\mathcal{B}_x = \mathcal{B}_{1m} \cos x + \mathcal{B}_{2m} \cos (x - 120) + \mathcal{B}_{3m} \cos (x + 120) \qquad (20)$$

If the currents i_1, i_2, and i_3 are constant the maximum flux densities $\mathcal{B}_{1m}$, $\mathcal{B}_{2m}$, and $\mathcal{B}_{3m}$ are also constant and the resultant field due to these currents is fixed in position relative to the armature conductors.

If the currents i_1, i_2, and i_3 vary sinusoidally with time, have the same maximum values, and are 120 time degrees out of phase it is permissible to assume that

$$i_1 = I_m \sin (\omega t - \theta_f)$$

$$i_2 = I_m \sin (\omega t - \theta_f - 120)$$

and

$$i_3 = I_m \sin (\omega t - \theta_f + 120) .$$

where ωt and θ_f are expressed in electrical time degrees. The angle θ_f corresponds to the angle θ_f in Fig. 65. (Note that the phase rotation of the currents is the same as the phase rotation of the generated e.m.f.'s.)

Since the major portion of the reluctance of the magnetic circuit is in the air-gap, it follows that

$$\mathcal{B}_{1m} = B \sin (\omega t - \theta_f)$$

$$\mathcal{B}_{2m} = B \sin (\omega t - \theta_f - 120)$$

$$\mathcal{B}_{3m} = B \sin (\omega t - \theta_f + 120)$$

where B represents the maximum flux density due to the maximum current (I_m) in a given phase.

Substituting these relations in equation (20) we obtain for the flux density at the point P the following relations:

$$\begin{aligned}\mathcal{B}_x &= B[\sin(\omega t-\theta_f)\cos x+\sin(\omega t-120-\theta_f)\cos(x-120)\\ &\quad +\sin(\omega t+120-\theta_f)\cos(x+120)] \qquad (21)\\ &= \frac{B}{2}\begin{bmatrix}\sin(\omega t+x-\theta_f)+\sin(\omega t-x-\theta_f)+\\ \sin(\omega t+x-240-\theta_f)+\sin(\omega t-x-\theta_f)+\\ \sin(\omega t+x+240-\theta_f)+\sin(\omega t-x-\theta_f)\end{bmatrix}\\ &= \tfrac{3}{2}B\sin(\omega t-x-\theta_f) \qquad (22)\end{aligned}$$

[Note that $\sin(y)+\sin(y-240)+\sin(y+240)=0$]

If we assume that point P travels around the armature conductors in the same direction as the main field and in such a way that in a given time the electrical space degrees covered are equal to the electrical time degrees of the current, and if we assume that at a time $t=0$ this point is located a distance x_0 from the center of coil c it follows that

$$\mathcal{B}_p = \tfrac{3}{2}B\sin[\omega t-(\omega t+x_0)-\theta_f]$$

or

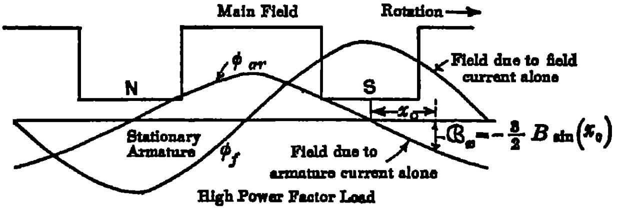
$$\mathcal{B}_p = -\tfrac{3}{2}B\sin(x_0+\theta_f) \qquad (23)$$

(Note that x_0 is also the distance between the center of the south pole S of the main field and the point P at the time $t=0$.)

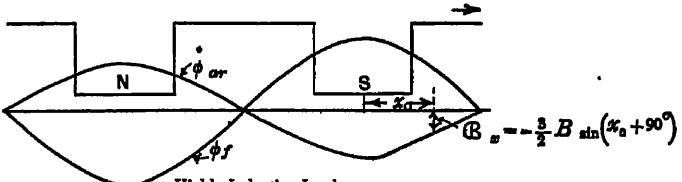
Equations (22) and (23) show that the flux density at a point in the air-gap of a three-phase generator is constant provided the point moves around the armature conductors at a speed $\omega=2\pi f$ electrical space radians per second and in the direction of rotation of the main field. This is equivalent to saying that the field produced by a balanced polyphase current in a symmetrically distributed polyphase armature winding rotates about the armature conductors at a constant speed and has a constant space wave form (on account of the armature slots this statement is not strictly true). In a three-phase machine the maximum resultant field density is $\tfrac{3}{2}B$ and is constant, where B is the maximum field density due to the maximum current in one of the phases.

Figures 67*a*, 67*b*, and 67*c* are the component wave forms of the field due to the field current and the armature currents for different values of the angle θ_f. The resultant field in the air-gap, neglecting saturation, is obtained by adding the two waves.

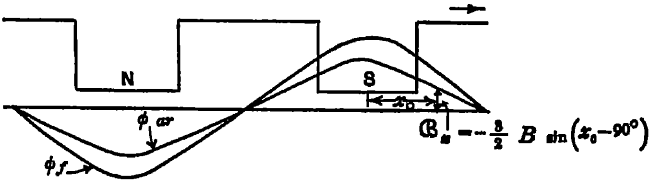
From Figs. 67*a*, 67*b*, and 67*c* it is apparent that the field produced by the armature current alone will either aid or oppose the field due to the field current alone depending on the internal power factor angle θ_f. If the angle $\theta_f=0$, corresponding to high internal power factor, the armature field (neglecting saturation) will neither magnetize nor demagnetize the main field poles. For a highly inductive load, θ_f positive, the armature field partly demagnetizes the main field, and for a condensive load, θ_f negative, the armature field partly magnetizes the main field. It is evident that in order to maintain the voltage of an alternator at its rated value, for a given armature current, it is necessary to have more


Main Field
Rotation →
ϕ_{ar}
N
S
Field due to field current alone
x_0
$\mathcal{B}_a = -\frac{3}{2} B \sin(x_0)$
Stationary Armature
ϕ_f
Field due to armature current alone

High Power Factor Load
$\theta_f = 0$
(*a*)



Highly Inductive Load
$\theta_f = 90°$
(*b*)



Highly Condensive Load
$\theta_f = -90°$
(*c*)

FIG. 67.—Component Field Forms of an Alternator Delivering Power to Three Kinds of Load

field current when the load is inductive and less field current when the load is condensive than that required for a unity power factor load.

84. Electromotive Force Generated in an Armature Coil due to the Armature Reaction Field.—In order to determine an expression for this e.m.f. it is necessary to obtain first an expression for the flux linking a coil. If we assume that coil *c* has a pitch of 180 electrical space degrees and that the effective length of the armature is *l*, the flux linking this coil is

$$\phi_c = l\int_{-90°}^{+90°} \mathfrak{B}_x dx \tag{24}$$

Substituting equation (22) in equation (24) and integrating with respect to x, we obtain for the flux linking the coil

$$\phi_c = \tfrac{3}{2}(2lB)\sin(\omega t - \theta_f) \tag{25}$$

The e.m.f. generated in the coil due to this flux taken positive in the same sense in which the current is taken positive is

$$e_{ar} = -N_c\frac{d\phi_c}{dt}10^{-8} = -\tfrac{3}{2}N_c(2lB)\omega 10^{-8}\cos(\omega t - \theta_f)$$

and the drop due to this e.m.f. taken positive in the same sense in which the current is taken as positive is

$$\begin{aligned} v_{ar} &= \tfrac{3}{2}N_c(2lB)\,\omega 10^{-8}\cos(\omega t - \theta_f) \\ &= \tfrac{3}{2}N_c(2lB)\,\omega 10^{-8}\sin(\omega t - \theta_f + 90) \end{aligned} \tag{26}$$

The r.m.s. value of this drop is

$$V_{(ar)coil} = \frac{\tfrac{3}{2}N_c(2lB)\omega 10^{-8}}{\sqrt{2}} \tag{27}$$

The phase armature reaction voltage V_{ar} is readily calculated [see equations (8)–(16)].

Since the coil under consideration belongs in phase 1 and the current in this phase is $i_1 = I_m \sin(\omega t - \theta_f)$ it follows from equation (26) that the voltage drop in this coil due to armature reaction leads the current by 90 degrees. This proves that the armature reaction drop V_{ar} in a phase of an alternator leads the current in the phase by 90 degrees and that it may be added directly to the leakage reactance drop $x_1 I_1$ as in Fig. 65.

85. Armature Phase Synchronous Reactance.—The arithmetical sum $(V_{ar}+x_1I_1)$ is called the synchronous reactance drop X_1I_1. The synchronous reactance may be defined by the relation

$$X_1=\frac{V_{ar}+x_1I_1}{I_1} \tag{28}$$

The reason for calling this the synchronous reactance is that the voltage V_{ar} is produced by a field cutting the armature conductors at a speed of $2\pi\left(\frac{p}{2}n\right)$ electrical space radians per sec., which represents the synchronous speed of the machine.

86. Synchronous Impedance.—The synchronous impedance of a phase of an alternator is its impedance when the current in the phase has a frequency of f cycles per second and the machine is driven at a speed of $n=\frac{2f}{p}$ revolutions per sec. The synchronous impedance may be calculated by the relation

$$Z_1=\sqrt{r_1{}^2+X_1{}^2} \tag{29}$$

where r_1 is the effective phase resistance and X_1 the phase synchronous reactance.

The effective resistance may be resolved into three components r'_1, r''_1, and r'''_1.

Where r'_1 = resistance of a phase as measured by direct-current;

$$r''_1=\frac{\text{Hysteresis and eddy-current loss in a phase due to the leakage flux in the iron surrounding the conductors}}{I_1{}^2}$$

and

$$r'''_1=\frac{\text{Eddy-current loss due to the flux in the copper conductors of a phase}}{I_1{}^2}$$

(I_1 is the phase armature current.)

In a-c. motors and generators the effective resistance of the windings is considerably greater than the d-c. resistance. Naturally the factor by which the d-c. resistance must be multiplied to obtain the effective resistance depends on the design of the machine. In most problems it may be assumed that $r_1=1.35r'_1$ (approximately).

The effective resistance is difficult to measure, but it so happens that extreme accuracy in its measurement is not required. The effective resistance is approximately one-tenth as large as the synchronous reactance and an error made in its measurement results in a much smaller error in the synchronous impedance.

87. A Method for Measuring the Synchronous Impedance of an Alternator.—The synchronous impedance of an alternator operating at a given frequency is not constant, but depends on the saturation of the field and armature magnetic circuits. In other words, the impedance depends on the field and the armature currents.

The best way for determining the synchronous impedance is to connect the armature of the alternator to a nearly zero lagging power factor load and then measure the frequency f, the armature phase terminal voltage V_1, the armature current I_1, the field current I_f, and the induced e.m.f. E'_1 in an armature phase due to the flux produced by the field current alone. (The armature phase e.m.f. E'_1 is called the excitation e.m.f. and may be obtained from the no-load saturation curve.)

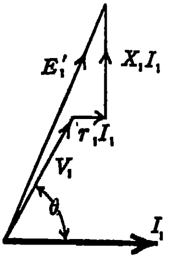


FIG. 68.—Voltage and Current Relations of an Alternator Operating at Nearly Zero Lagging Power Factor

The vector diagram in Fig. 68 gives the voltage and current relations in a phase of an alternator when it is operated at approximately zero power factor. Under these conditions the synchronous impedance drop is nearly equal to the arithmetical difference between the excitation voltage and the terminal voltage. That is

$$Z_1I_1 = E'_1 - V_1 \quad \text{(arithmetical difference)}$$

$$\therefore \ Z_1 = \frac{E'_1 - V_1}{I_1} \quad \text{(approximately)}$$

88. Methods for Determining the Voltage Regulation of an Alternator.—The per cent voltage regulation of an alternator is obtained by the relation

$$\text{Per cent regulation} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} 100 \qquad (30)$$

Where V_{load} is the phase terminal voltage when the load is connected and $V_{\text{no-load}}$ is the phase terminal voltage at no-load. Both of these voltages must be measured at the same frequency and at the same field current.

The voltage regulation for a given load may be determined, (1) by actually loading the machine or (2) by calculation from curves obtained from no-load tests. The first method is applicable to small machines only, while the second method may be applied to machines of any size. (See A.I.E.E. Standardization Rules in regard to the methods for determining the voltage regulation of alternators.)

Method (1) is self-evident and no further explanation of this method is required. Method (2) is based on the synchronous impedance determined from the no-load and the zero power factor saturation curves.

The no-load saturation curve is obtained by operating the machine at rated speed and measuring the no-load phase voltage and the field current. The zero power factor saturation curve is obtained by loading the alternator on a highly lagging power factor (idle running under excited synchronous motor) load and at rated speed and rated armature current measure the armature phase terminal voltage and the field current.

If the curves are plotted on the same sheet and to the same scale, using voltages for ordinates and field currents for abscissas, the zero power factor curve should on account of the demagnetizing action of the armature be located under the no-load saturation curve. Fig. 69 shows typical curves obtained in this way. These curves show that for a given armature current the synchronous impedance drop is far from constant; it depends on the field current. Thus, the drop $B'C'$ when the field current is small is much greater than the drop BC when the field is saturated.

For zero power factor rated kv-a. load and for rated phase voltage the line AB represents rated voltage, AC the excitation or no-load voltage, and BC the synchronous impedance drop. For zero power factor rated kv-a. load.

$$\text{Per cent regulation} = \frac{AC - AB}{AB} \times 100$$

To determine the regulation at rated voltage, frequency, and load for any lagging power factor it is necessary to construct the diagram shown in Fig. 70. This diagram is obtained by constructing first the impedance drop triangle BCD. The line BD represents the effective resistance drop, and *BC* the synchronous impedance drop. The line *BC* is equal to the distance *BC* in Fig. 69 and is so drawn that *CD* is perpendicular to *BD*. The line *AB* is equal to *AB* in Fig. 69 and is so drawn that it makes an angle θ with XX'. The quantity $\cos\theta$ represents the power factor of the load. The line *AC* is the no-load voltage. For this particular load the per cent voltage regulation is $\frac{AC-AB}{AB}100$. The regulation in this case is positive but is considerably less than for zero lagging power load. For a rated kv-a. condensive load the voltage regulation is obtained from Fig. 71. In this figure the triangle *BCD* corresponds in every way to the triangle *BCD* in Fig. 70. The per cent regulation $\frac{AC-AB}{AB}100$ may be positive, zero, or negative. A negative regulation means that the full-load voltage,

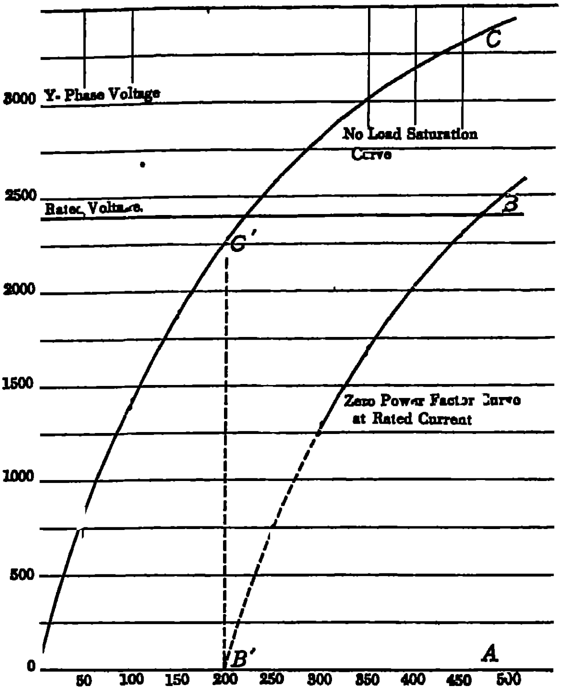


FIG. 69.—No-load and Zero Power Factor Saturation Curves of an Alternator

on account of the magnetizing action of the armature currents is higher than the no-load voltage.

The voltage regulation of large alternators may be as high as 30 or 40 per cent. Small voltage regulations are not desired on account of the forces associated with the current when the machine is accidently short-circuited; a small regulation means a small synchronous impedance and a terrific short-circuit current. In large generating stations the generators are operated in parallel and the load on any one machine does not fluctuate over wide limits. For this reason automatic voltage regulators may be used to control the terminal voltage and poor inherent regulation in the alternators themselves is no serious matter. On the other hand, in small generating stations where power is furnished to lighting circuits and to large motors, good inherent voltage regulation of the alternators is of importance.

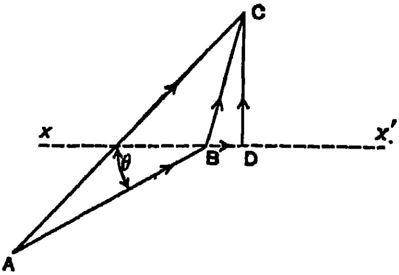


Fig. 70.—Diagram for Determining the Voltage Regulation of an Alternator at Lagging Power Factor

89. Current-limiting Reactors (Fig. 72).—The performance of large a-c. generators under short-circuit conditions is of extreme importance. If a generator is short-circuited at its terminals the machine is loaded on its own inductive armature winding. The initial value of the short-circuit current depends on the excitation voltage and the initial value of the impedance of the armature winding. The armature reaction field due to the short-circuit current requires time to become effective in demagnetizing the main field and the result of this lag is that for an instant the short-circuit current may be roughly eight times the rated current of the machine. The steady state value of the short-circuit current is roughly three times the rated current of the machine. The destructive effect of these currents is readily realized when

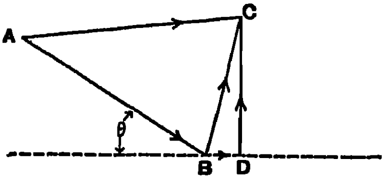
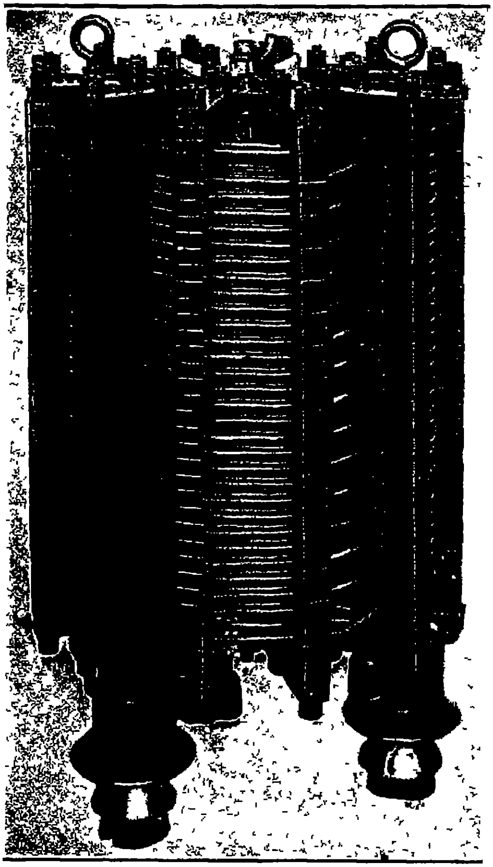


Fig. 71.—Diagram for Determining the Voltage Regulation of an Alternator at Leading Power Factor

we recall that the mechanical forces between conductors as well as the heating of the windings depend on the square of the currents in the windings and may be many times normal.

The preceding discussion applies to infrequent short-circuits which take place at the terminals of the machine. Ordinarily short-circuits happen at some point in the transmission system located away from the generator. To overcome the destructive effects of excessive currents, current-limiting reactors are connected in each armature lead of the generator. The drop across each reactor at rated armature current is roughly 15 per cent of the rated phase voltage of the generator. It is evident that with the reactors connected, the short-circuit currents are greatly reduced.



(By the courtesy of the Westinghouse Co.)

Fig. 72.—Current-limiting Reactor

90. Tirrill Automatic Voltage Regulator.—The voltage required at the terminals of an alternator to maintain constant and normal voltage at the load is, on account of the transmission line drop, a function of the armature current. An automatic voltage regulator must maintain constant voltage at a given point on the transmission system and must therefore change the terminal voltage of the alternator as the transmission line drop changes.

Several types of automatic regulators are used and many improvements have been made on the simplified Tirrill regulator shown in Fig. 73.

All automatic generator voltage regulators function by short-circuiting, at intervals, a portion of the field rheostat of the exciter in the generator field circuit. The duration of the time which the field resistance is cut out determines the terminal voltage of the exciter and therefore the terminal voltage of the a-c. generator.

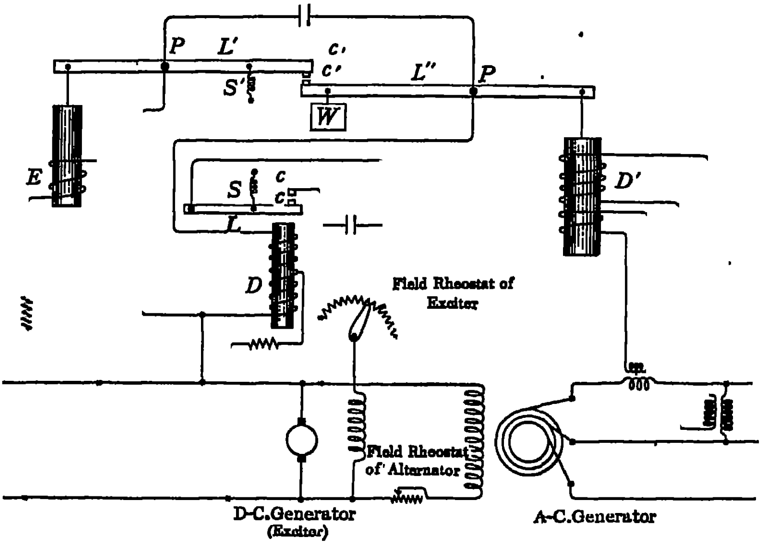


Fig. 73.—Automatic Voltage Regulator for an Alternator

The regulator shown in Fig. 73 has two sets of contact points. The contact points *cc* are used for short-circuiting a portion of the field rheostat. The location of the contact points *c'c'* depends on the exciter voltage, the alternator voltage, and the armature current of the alternator.

The contacts at *cc* and *c'c'* are made or broken at approximately the same instants. When the circuit through *c'c'* is open, the bottom half of coil *D* is excited and lever *L* is pulled down, opening the circuit through *cc*. When the circuit through *c'c'* is completed, both halves of coil *D* are excited in opposite directions, the magnetic field produced by the coil is zero, the lever *L* is released and the circuit is completed through *cc*.

In order to explain the complete action of the regulator let us first assume that the armature current and terminal voltage of the alternator are constant. Lever L'' will then remain in a fixed position, for the windings D' produce a constant magnetic force. If under these conditions the exciter voltage decreases, the current in coil E becomes less, the contacts $c'c'$ and cc are closed, and the field rheostat of the exciter is short-circuited. The decreased field resistance raises the exciter voltage, the current in coil E increases, and the contacts $c'c'$ and cc are opened. In actual operation the contacts $c'c'$ and cc vibrate continuously.

In order to make the excitation of the alternator a function of the load a device is required which makes the closing of contacts $c'c'$ easier when the load on the alternator increases. This is accomplished by coils D' and lever L''. One of the coils on D' is connected to a potential transformer and the other to a current transformer. These coils tend to produce flux in opposite directions and in such a way that when the alternator armature current increases the lower contact $c'c'$ moves upward. The ease at which contact is made at $c'c'$ is increased and the alternator voltage assumes a higher value.

91. Efficiency of an Alternator.—By the efficiency of a generator is meant the ratio of the useful output to the input. Unless otherwise stated the efficiency specified for a given generator is the efficiency at rated armature current, voltage, frequency, power factor, and at the standard reference temperature of 75° C.

The efficiency of small generators may be obtained from simultaneous measurements of the output and the input.

The efficiency of large generators is determined by the relation

$$\text{Eff.}=\frac{\text{Output}}{\text{Output}+\text{losses}}=\frac{\text{Input}-\text{losses}}{\text{Input}} \qquad (31)$$

The output or input may be assumed, but the losses must be determined by, (1) measuring most of the losses by direct test and assigning conventional values to those which cannot be determined accurately by test, or (2) by measuring all losses using conventional methods of test. The efficiency calculated from losses obtained in whole or in part by conventional methods is called the conventional efficiency.

The losses in an a-c. generator are as follows:

1. Copper losses in the armature and field windings. These losses include brush contact rI^2 losses and field rheostat losses.
2. Friction and windage losses.
3. Core losses in the magnetic circuits of the machine.
4. Stray-load losses consisting of iron losses and eddy-current losses in the conductors and other metal parts of the machine which vary with load or with magnetic saturation.
5. Dielectric losses in the insulation.

92. Determination of the Losses in an Alternator.—

1. The copper losses at the standard reference temperature of 75° C. are calculated from resistances of the armature and field windings obtained by ordinary measurements of the d-c. resistances.
2. The friction and windage losses are determined by measuring the power input necessary to drive the machine at rated speed with both the armature and field circuits open.
3. The core losses and losses in the armature conductors and insulation are obtained by the difference between the power input at rated speed with the field excited and the armature circuit open and the input obtained in (2). The excitation voltage in this test should equal the rated voltage of the generator + the resistance drop in the armature at rated armature current.
4. The stray-load losses of a polyphase generator are determined by operating the machine on short-circuit at rated armature current and speed. The input under these conditions less the armature copper loss and the friction and windage loss represent the stray load losses.

The losses in an exciter or a ventilating blower when supplying a single generator are charged to the generator. If the exciter and the blower supply several generators these losses are charged against the plant as a whole.

93. Rating of an Alternator.—The kv-a. rating of an alternator is determined by the safe terminal voltage and the safe terminal currrent. The operating voltage is determined by the insulation, the air-gap flux, the speed, the number of turns in the armature winding and the voltage drop in the armature. The armature current is limited by the heating in the armature and the field.

The current required in the field to produce rated armature terminal voltage depends on the armature current and the power factor of the load. Lagging power factor loads require more field current to maintain rated voltage than unity or leading power factor loads. The kv-a. rating of an alternator on account of field heating, is less for inductive loads than for condensive loads. A three-phase alternator rated for 37.5 kv-a. at 110 volts and 80 per cent lagging power factor means that the field and armature windings are so designed that the machine is able to supply, without overheating, an inductive load requiring 110 volts and 196.5 amperes at 80 per cent lagging power factor.

94. Values for the Losses and the Voltage Regulation of a 10,000 kv-a. Alternator.—

RATING

10,000 kv-a.; 4160 volts; 3-phase; 60-cycles;
80 per cent lagging power factor.

LOSSES AT RATED LOAD (75°C.)

	Kilowatts
RI^2 losses in stator (total)	= 32.50
RI^2 losses in rotor winding	= 46.00
Stray-load losses	= 140.00
Core losses	= 130.20
Friction and windage	= 145.00

VOLTAGE REGULATION AT RATED TERMINAL VOLTAGE AND RATED ARMATURE CURRENT

	Per Cent
Reg. at zero lagging power factor	= 41.0
Reg. at 0.8 lagging power factor	= 32.5
Reg. at unity power factor	= 19.0

PROBLEMS

1. An alternator has 8 poles and is driven at a speed of 1350 r.p.m. Calculate the frequency of the voltage generated.

2. The armature of a 6-pole, three-phase alternator has a double layer winding. The armature has 72 slots. There are six coil groups per phase and four coils per coil group (corresponding sides of the coils belonging to a coil

group are placed in adjacent slots). The coils in a group and the coil groups belonging to a phase are all connected in series. The coil pitch is 120 electrical space degrees. Draw: (*a*) an armature winding diagram; and (*b*) a section through the armature showing the location of the conductors belonging to a given phase.

3. A two-phase alternator has 4 poles, operates at a speed of 1800 r.p.m., and has 600 turns in series per phase. The flux linking each turn varies sinusoidally from a positive maximum value of 1,200,000 lines to a negative maximum value of the same number of lines. The e.m.f.'s in the turns are in phase. Calculate: (*a*) the frequency of the induced e.m.f.; and (*b*) the r.m.s. value of the induced voltage between the armature terminals.

4. Each phase of a 6-pole, Y-connected alternator consists of six coil groups connected in series. Each coil group has three coils connected in series. The adjacent coil sides in a group are placed in adjacent slots 20 electrical space degrees apart. Each coil has 10 turns. The field wave form has a fundamental of 1,500,000 lines and a third harmonic of 200,000 lines. The flux densities, due to the fundamental and the third harmonic in the space distribution of the flux have their maximum values at the center of a given pole face and the two fields have opposite directions at this point. The alternator is driven at a speed of 1200 r.p.m. Calculate (*a*) the Y-phase and terminal e.m.f.'s of the alternator when full-pitch coils are used; and (*b*) the Y-phase and terminal e.m.f.'s when the pitch of the coils is $\frac{2}{3}\pi$. (*c*) What are the shapes of the voltage waves in parts (*a*) and (*b*)?

5. A single-phase alternator with a distributed mesh armature winding is rated at 7.5 kv-a., 110 volts. Calculate the kv-a. and voltage rating of this machine when connected for three-phase operation. (Assume the armature copper loss the same for both connections.)

6. A single-phase alternator delivers 7 kv-a. at 125 volts to, (1) a condensive load, (2) a unity power factor load, and (3) an inductive load. The field currents required to maintain 120 volts for the different loads are 1.2, 1.6, and 1.8 amperes respectively. Explain. Which load gives the highest efficiency for the generator?

7. The following things are known about a 5500-kv-a., 6600-volt, 60-cycle, three-phase generator:

Effective resistance of armature (temp. 75° C.) = 0.10 ohm per Y-phase.

Synchronous impedance of armature (temp. 75° C.) = 1.6 ohm per Y-phase.

The generator delivers rated kv-a. at rated voltage and at, (1) 85 per cent lagging power factor, (2) unity power factor, and at (3) 85 per cent leading power factor. (*a*) Calculate the Y-phase excitation voltage (the voltage due to the flux produced by the main field current acting alone) required for each load. (*b*) Construct complete vector diagrams showing the voltage and current relations for each load.

8. The following data were obtained on a 15-kv-a., 220-volt, 60-cycle, three-phase generator:

NO-LOAD SATURATION CURVE

Field Current, Amps.	Terminal Voltage, Volts	Frequency
2	80	60
4	146	60
6	198	60
10	264	60
12	284	60
14	296	60
16	304	60

ZERO POWER FACTOR SATURATION CURVE, ARMATURE CURRENT, 39.4 AMPS.; TEMPERATURE, 75° C.

Field Current, Amps.	Terminal Voltage, Volts	Frequency
6.9	0	60
8.0	40	60
10.0	103	60
12.0	158	60
14.0	199	60
16.0	229	60
18.0	251	60

D-c. resistance measured between armature terminals (temp. 34° C.) = 0.234 ohm.

Ratio $R_{a\text{-}c.}/R_{d\text{-}c.} = 1.34$, at 75° C.

(*a*) Plot the no-load and the zero power factor saturation curves on the same sheet of paper. (*b*) Calculate: (1) the effective Y-phase resistance; (2) the Y-phase synchronous impedance at rated terminal voltage, rated current, and at zero lagging power factor; and (3) the Y-phase synchronous reactance for the same conditions as specified under (2). (All values obtained in part (*b*) must be referred to the standard reference temperature of 75° C.)

9. (*a*) The generator specified in Problem 8 supplies rated kv-a. at rated voltage to an 83 per cent lagging power factor load. Calculate the voltage regulation of the alternator at 75° C. Use one of the methods outlined in the A.I.E.E. Standardization Rules.

(*b*) Repeat *a* for a unity power factor load and for an 83 per cent leading power factor load.

10. Estimate the zero power factor saturation curve in Problem 8 by the method outlined in the A.I.E.E. Standardization Rules.

11. The alternator in Article 94 supplies rated kv-a. at rated voltage to an 80 per cent lagging power factor load. Calculate its efficiency under these conditions.

CHAPTER VII

OPERATION OF ALTERNATORS IN PARALLEL

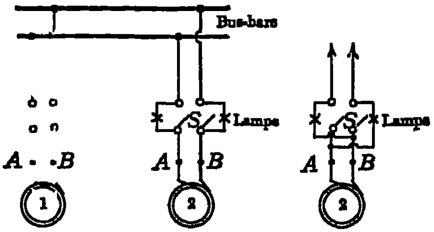
95. Reasons for Operating Generators and Generating Stations in Parallel.—The present limit of the capacity of a single armature synchronous generator is approximately 60,000 kv-a. For this reason a generating station which, for example, supplies a maximum load of 300,000 kv-a. requires that at least five of these generators be operated in parallel.

On account of the economy involved in present-day high-voltage transmission it is advantageous to operate distant stations in parallel. This makes it possible to locate the power stations near the energy resources. Interconnection of many distant stations insures continuity of service and permits the utilization of energy which would otherwise be wasted.

96. Synchronizing of Alternators.—In order to operate alternators in parallel the machines must have equal frequencies and the e.m.f.'s in corresponding phases must be in opposition with reference to the local circuit formed by the bus-bars and the armatures. The actual process of determining when these conditions are satisfied is called synchronizing. The alternator which is to be synchronized is connected through a switch that may be closed when the proper conditions are satisfied. Synchronism is determined by the use of lamps or synchroscopes.

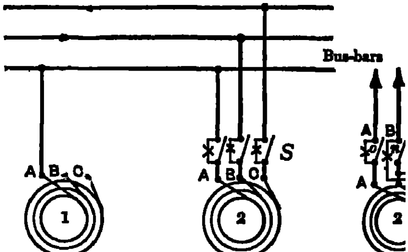
Figure 74*a* illustrates how two lamps may be connected for determining synchronism of two low-voltage single-phase alternators. When the e.m.f. of alternator No. 1 (in the sense *A* to *B*) is in phase and equal to the e.m.f. of alternator No. 2 (in the sense *A* to *B*) no voltage exists across the lamps, the lamps are dark, and the synchronizing switch *S* may be closed. Any torque tending to throw the machines out of synchronism is accompanied by a reverse torque tending to hold them in synchronism. To avoid closing the synchronizing switch when one of the lamps is burned out the connection shown in Fig. 74*b* is often used. Synchronism in this case is indicated when the l have maximum brilliancy.

Figures 75*a* and 75*b* illustrate two methods of connecting l for synchronizing two three-phase low-voltage alternators synchronizing any polyphase alternator it is necessary tha phase rotation of the ages (the sequence in the voltages in the p reach their maxi values) of the inco machine be the sar that of the voltag the bus-bars to whic machine is to be cor ed. The phase rot is correct when the l connected as in Fig go on or off at the same time. Synchronism in this c indicated when all the lamps are dark. Another method, k as the Siemens and Halske connection of synchronizing lan shown in Fig. 75*b*. In this case, when synchronism is approa the lamps *m*, *n*, and *o* reach their maximum brilliancies in a sequence which depends on the speed of the incoming machine. For the particular connection shown, synchronism is indicated when lamps *m* and *n* are equally bright and lamp *o* is dark. In order to synchronize high-voltage machines, synchronizing lamps and phase meters mu supplied through potential transformers.



(*a*) Lamps Dark (*b*) Lamps Bright

FIG. 74.—Methods for Determining Synchronism of Two Alternators



Lamps Bright (*a*) Lamp and lamps equally l (*b*

FIG. 75.—Methods for Synchronizing Three-phase Generators

97. The Phase Meter Used as a Synchroscope.—One fo phase-meter shown in Fig. 76 is provided with three coils, and *c*. Coils *a* and *b* are mechanically fastened together ar free to rotate in either direction. Coil *a* is connected in with a resistance and coil b in series with an inductance to the terminals of one of the alternators. The electrical connections of these coils are so made that a current entering the connecting lead (1) results in two opposed torques. Coil c is connected in series with a resistance to the terminals of the other alternator.

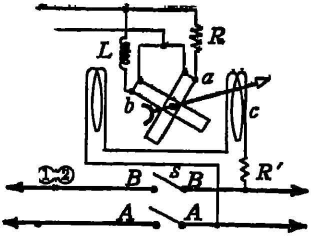


Fig. 76.—Phase Meter Used as a Synchroscope

If the voltages on both sides of the synchronizing switch taken positive in the sense A to B are in phase, the currents in coils a and c will be in phase and the current in coil b will be nearly 90 time degrees behind the currents in these coils. Under these conditions the average torque exerted on b due to the magnetic reaction between b and c is zero and the reaction between c and a is sufficient to turn the two coils until the axis of a coincides with the axis of c. If the voltages on both sides of the synchronizing switch are out of phase, the average torques exerted on a and b are opposite and the coils will assume a position where the average torques acting on them balance. The position which the coils assume is an indication of the phase relation between the voltages on the two sides of the switch. A difference in the frequencies of the two generators results in a continuous change in the phase relations and the coils a and b rotate in a direction depending on which machine has the higher frequency.

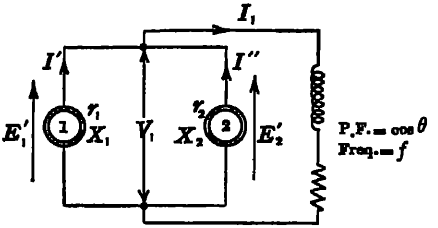


Fig. 77.—Connections of Corresponding Phases of Two Alternators in Parallel

98. Theory of Operation of Two Alternators in Parallel.—The corresponding phases of alternators No. 1 and No. 2 are connected as shown in Fig. 77. The following analysis is based on the supposition that the following things are known:

(*a*) The constants of the alternators.

(*b*) The total load connected to the bus-bars.

(*c*) The load voltage, load current, and load power factor measured at the bus-bars.

(*d*) The excitation e.m.f. and power output of one of the alternators. (The analysis shows what adjustments of the field currents of the alternators and governors of the prime movers are required to obtain certain definite operating conditions.)

Let V_1 = phase voltage of connected load measured at bus-bars.
I_1 = phase current of load connected to bus-bars;
$\cos\theta$ = phase power factor of load connected to bus-bars measured at the bus-bars;
P_1 = phase power of alternator No. 1 measured at its terminals;
r_1 = phase effective resistance of alternator No. 1;
r_2 = phase effective resistance of alternator No. 2;
X_1 = phase synchronous reactance of alternator No. 1;
X_2 = phase synchronous reactance of alternator No. 2;
E'_1 = phase excitation e.m.f. of alternator No. 1;
E'_2 = phase excitation e.m.f. of alternator No. 2;
I' = phase current of alternator No. 1;
I'' = phase current of alternator No. 2;
$\cos\theta'$ = phase power factor of alternator No. 1;
$\cos\theta''$ = phase power factor of alternator No. 2.

The phase terminal voltages of the alternators on account of the interconnections are equal, and therefore

$$\dot{V}_1 = \dot{E}'_1 - (r_1 + jX_1)\dot{I}' \tag{1}$$

$$\dot{V}_1 = \dot{E}'_2 - (r_2 + jX_2)\dot{I}'' \tag{2}$$

The total current output may be expressed by the relation

$$\dot{I}_1 = \dot{I}' + \dot{I}'' \tag{3}$$

Equation (1) may be written

$$\dot{V}_1 = \dot{E}'_1 - \dot{Z}_1(I'\cos\theta' \pm jI'\sin\theta') \tag{4}$$

where all vectors are referred to the voltage. If we assume that V_1, P_1, and $\cos\theta'$ are known, it is possible to calculate the power component of the current output of alternator No. 1 as follows:

$$P_1 = V_1 I' \cos\theta'$$

or

$$I'\cos\theta' = \frac{P_1}{V_1} \tag{5}$$

If we assume that E'_1 and Z_1 are also known it is possible to calculate the reactive component $I' \sin \theta'$ of the current output. From equations (4) and (5) it follows that

$$\dot{V}_1 = \dot{E}'_1 - \dot{Z}_1\left(\frac{P_1}{V_1} - jI' \sin \theta'\right) \tag{6}$$

where Z_1 is the synchronous impedance of a phase of alternator No. 1 and θ' is considered positive when the phase voltage V_1 leads the corresponding phase current I'.

Solving equation (6), we obtain

$$I' \sin \theta' = \frac{\dot{V}_1 - \dot{E}'_1 + \dot{Z}_1 \frac{P_1}{V_1}}{j\dot{Z}_1} \tag{7}$$

The reactive component $I' \sin \theta'$ is 90 degrees displaced from the phase voltage V_1.

Equation (7) shows that for a given output, load voltage, and synchronous impedance, the reactive component of the current output of a given alternator depends on its excitation voltage.

The total current output of alternator No. 1 is

$$I' = \sqrt{(I' \cos \theta')^2 + (I' \sin \theta')^2} \tag{8}$$

The in-phase component of the current output of alternator No. 2 may be calculated by the relation

$$V_1 I'' \cos \theta'' + V_1 I' \cos \theta' = V_1 I_1 \cos \theta \tag{9}$$

and the reactive component of the current output of this machine may be calculated by the relation

$$I'' \sin \theta'' + I' \sin \theta' = I_1 \sin \theta \tag{10}$$

The total current output of alternator No. 2 is

$$I'' = \sqrt{(I'' \cos \theta'')^2 + (I'' \sin \theta'')^2} \tag{11}$$

By substituting the values $I'' \cos \theta''$ and $I'' \sin \theta''$ obtained from equations (9) and (10) in equation (2) we obtain for the excitation e.m.f. of alternator No 2 the relation

$$\dot{E}'_2 = \dot{V}_1 + \dot{Z}_2(I'' \cos \theta'' - jI'' \sin \theta'') \tag{12}$$

99. Vector Diagram Representing Equations (3), (6), and (12) (see Fig. 78).—The vector diagram shows that for a given phase terminal voltage V_1 and for a given phase power output P_1 (constant value for $I' \cos \theta'$), the power factor of alternator No. 1 depends on its excitation voltage. A reduction of the excitation voltage from the value E'_1 to (E'_1) results in a reversal of the voltage drop due to the reactive component of the current, i.e., a reversal of the drop $Z_1 I' \sin \theta'$, and therefore a reversal of the current $I' \sin \theta'$. A similar method may be employed to show that the power factor of alternator No. 2 depends largely on its excitation e.m.f.

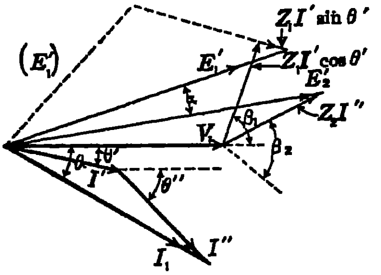


FIG. 78.—Vector Diagram for Two Alternators in Parallel

100. Adjustments Necessary to Alter the Power and Power Factor of a Given Alternator Operating in Parallel with Others.—The performance of an a-c. motor or a transformer depends on the impressed frequency. For this reason alternators must operate at constant frequency and therefore must be driven at constant speed regardless of the electrical load on the machine.

The power output of a steam engine or steam turbine is determined by the amount of steam admitted to the engine. A given engine connected to a given boiler and delivering H horse power at a speed of N r.p.m. for M pounds of steam delivers H' horse power at a speed of N r.p.m. when M' pounds of steam are admitted to the engine. To maintain constant speed the mechanical load on the engine must be increased when the amount of steam admitted to the engine is increased and vice versa. If the given engine drives an alternator which is operated in parallel with others it must operate at constant speed and the alternator output can only be changed by changing the driving torque of the engine; i.e., by changing the governor setting of the engine.

If alternators No. 1 and No. 2 in Fig. 77 are of the same rating and it is desired to increase the load on No. 1 and reduce that on No. 2, without changing the frequency, it is necessary to tighten the governor of the prime mover driving No. 1 and to loosen the governor of the prime mover driving alternator No. 2.

If the rating of alternator No. 2 is much greater than the rating of alternator No. 1, a load adjustment on No. 1 may be made by adjusting the governor of No. 1 only.

For a constant speed and a constant terminal voltage a change in the excitation voltage (a change in the field current) of either alternator will not materially change the power output of the alternators. This is evident when we keep in mind that the excitation voltages of the alternators have absolutely no control over the driving torques developed by the prime movers. A change in excitation voltages changes the reactive components of the currents (the components $I' \sin \theta'$ and $I'' \sin \theta''$) and therefore changes the resultant currents I' and I'', the power factors $\cos \theta'$ and $\cos \theta''$, and the copper losses of the alternators. An increase of the copper loss for a given available driving torque results in only a small reduction of the power output at the terminals. A radical change in excitation voltages may cause excessive heating in the field and armature circuits.

101. Conditions Necessary for Most Economical Operation of Alternators in Parallel.—For most economical operation of alternators in parallel, the following conditions should be satisfied:

(1) The arithmetical sum of the kv-a. outputs of the alternators should equal the resultant kv-a. of the connected loads.

(2) The current outputs of the alternators should bear the same ratio as the kv-a. ratings of the machines.

(3) The resistances of the armature windings should be proportioned in the inverse ratio of the current outputs.

If the first and second conditions are not satisfied it is impossible to utilize the full kv-a. ratings of the two generators and it may be necessary to operate a third machine, although the total connected load does not warrant this. In order to satisfy the first condition it is necessary that $\cos \theta' = \cos \theta'' = \cos \theta$; i.e., the power factors of the generators must be equal and therefore must be the same as the power factor of the connected load. Both of these conditions may be obtained by properly changing the components of the current outputs of the generators which may always be accomplished by adjusting the field excitations of the generators and the driving torques of the prime movers.

If the third condition is not satisfied the copper losses in the armatures are greater than necessary. The design of the generators determines whether this condition can or cannot be obtained.

102. Mathematical Summary of Circuit Conditions Necessary to Obtain Economical Parallel Operation.—

Let $\beta_1=\tan^{-1}\dfrac{X_1}{r_1}$

$\beta_2=\tan^{-1}\dfrac{X_2}{r_2}$

and α = electrical time angle between the vectors E'_1 and E'_2 in Fig. 78. [In two identical alternators, α is numerically equal to the electrical space angle between the centers of corresponding poles on the alternators (see Fig. 79).]

If $E'_1=E'_2$ (condition for minimum excitation loss)

and

$$\alpha=0$$

then

$\underset{\cdot}{Z}_1\underset{\cdot}{I}'=\underset{\cdot}{Z}_2\underset{\cdot}{I}''$ [see Fig. 78 or equations (1) and (2)]

and

$$Z_1I'\epsilon^{j(\beta_1-\theta')}=Z_2I''\epsilon^{j(\beta_2-\theta'')} \qquad (13)$$

Again if

$$\frac{X_1}{r_1}=\frac{X_2}{r_2}$$

then

$$\beta_1=\beta_2 \qquad (14)$$

and by substituting this value in (13) we obtain

$$\theta'=\theta''$$

or $\cos\theta'=\cos\theta''$ (condition necessary to utilize the total rating of the alternators) (15)

and

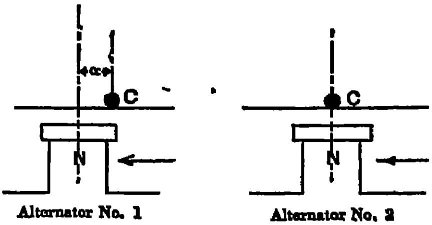
$$\frac{I'}{I''}=\frac{Z_2}{Z_1}=\frac{\sqrt{r_2^2+X_2^2}}{\sqrt{r_1^2+X_1^2}}=\frac{r_2\sqrt{1+\tan\beta_2^2}}{r_1\sqrt{1+\tan\beta_1^2}}=\frac{r_2}{r_1}$$

Since the terminal voltages of the machines are equal it follows that

$\dfrac{\text{KV-A. rating of No 1}}{\text{KV-A. rating of No. 2}}=\dfrac{r_2}{r_1}$ (condition for minimum armature copper loss) (16)

It should be observed that if the impedance angles β_1 and β_2 are not equal it will still be possible to satisfy relation (15); i.e., kv-a. output, of No. 1 + kv-a. output of No. 2 = total kv-a. output, but in general E'_1 will not be equal to E'_2 and α will not be equal to zero.

103. Torques Acting on Rotor; Synchronizing Current.—The angle α between the excitation voltages E'_1 and E'_2 is a measure of the relative positions of the rotors (usually the fields) of the alternators at any instant (see Fig. 79), for the difference in phase between the two excitation voltages at any instant depends upon the positions of corresponding poles relative to corresponding armature conductors. In general any change in the mechanical input; i.e., any change in the driving torque of either machine, will cause the angle α to change. If the driving torque of machine No. 1 is increased, the in-phase component of its armature current is increased and E'_1 will move ahead of its former position; that is, the rotor of machine No. 1 will slip forward with respect to the rotor of machine No. 2.



CC.Corresponding Armature Conductors

FIG. 79.—The Electrical Space Angle α

As long as the rotor of each machine is revolving with a perfectly uniform speed the driving torque of the prime mover is exactly balanced by an opposing torque due to the electromagnetic attraction between the field and the armature current. The driving torque is in the direction of rotation and the opposing torque due to the electromagnetic forces, which may be called the "electromagnetic torque," is in the direction opposite to the direction of rotation. The latter torque is equal to the electrical power developed in the armature divided by the speed of rotation of the rotor, for power is equal to the product of speed by torque. Neglecting the copper losses in the armature, the electromagnetic torque is then proportional to the electric power output, since the speed is constant. Any change in the in-phase component of the current output therefore means a corresponding change in the electromagnetic torque. Should the rotor of machine No. 1 at any instant fall behind the position corresponding to an exact balance between the driving and electromagnetic torques, the in-phase component of the current I' becomes less, the torque opposing the driving torque is diminished, and the unbalanced portion of the driving torque is available to pull the rotor back into its normal position (i.e., into the position of exact balance between the driving and electromagnetic torque). Again, should the rotor of machine No. 1 slip ahead of its normal position, the electromagnetic torque (which is always in a direction opposite to that of the rotation) will be in excess of the driving torque, since the in-phase component of the current I' will be in excess of its normal value, and the net torque acting on the rotor will be in such a direction as to pull it back into its normal position.

Since the torque which holds the rotor in step, or pulls it back into step should it tend to fall out of step, depends upon the change of the in-phase component of the current I', the change of the in-phase component of the current output may be called the "synchronizing current."

104. Hunting.—As pointed out in the preceding article, when the rotor of either machine slips backward or forward from its normal position, due to any cause whatever, a restoring torque is set up, this torque acting in a direction opposite to that of the displacement. The restoring torque will not only pull the rotor back into its equilibrium position, but will give it a certain rotational velocity with respect to its equilibrium position, and therefore, due to its inertia, the rotor will swing beyond its equilibrium position until the restoring torque, acting now in the reverse direction (but always toward the equilibrium position), causes it to swing back again. In short, the rotor will act exactly like a torsional pendulum and will oscillate back and forth about its equilibrium position. When such hunting starts, additional hysteresis and eddy-current losses will be produced in the pole faces, and in any damping grids in the pole faces (due to the change in position of the resultant field with respect to the main field poles) and these energy losses will damp out the oscillations unless the cause which starts the hunting is recurrent. In general, hunting of one alternator will cause the other alternators in parallel with it to hunt, unless the capacity of the latter is large compared with the capacity of the one which is hunting.

When the prime mover is a reciporcating engine the driving torque is not uniform throughout each revolution, but pulsates to a greater or lesser extent, depending upon the type of engine. The driving torque of most multiple cylinder engines never falls to zero, but varies between maximum values. Should the pulsations in the driving torque have a period equal to that of the natural period of the rotor oscillations, the amplitude of the oscillations may become excessive, so great in fact as to cause the rotor to fall out of step.

Even though the period of pulsation in the driving torque should not coincide with the natural period of the rotor oscillations, hunting will occur, but the system will be stable provided the rotor does not swing beyond the position corresponding to its maximum output.

Hunting may be caused by any periodic variation in the circuit such as may be produced by regular variations in the load, but it is seldom that the periodic variations in a load will coincide with the natural period of the alternators.

Hunting in most cases is successfully diminished by a short-circuited winding embedded in the pole faces of the alternators which are operated in parallel. This pole-face or damping winding often takes the form of a squirrel cage made of rectangular conductors short-circuited by end rings. The action of the pole-face windings is explained in detail in the chapter on synchronous motors.

105. Natural Period of Oscillation of Rotor.—Consider the case when the alternator No. 1 in which hunting occurs is in parallel with one or more alternators No. 2 which have a sufficient power capacity not to be affected by the alternator which is hunting. Under these conditions changes in the electrical output of the given alternator as its rotor oscillates will not affect appreciably the bus-bar voltage or speed of the system. The impedance Z_2 of the alternator (or group of alternators) in parallel with alternator No. 1 will be small compared with Z_1, and therefore for a given load on the bus-bars, the vector E'_2 in Fig. 80 may, at least for small changes in the current output, be considered as

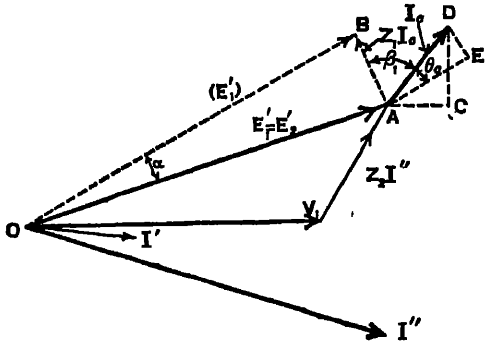


FIG. 80.—Diagram Showing Change in Current Output when an Alternator Hunts

fixed in magnitude and direction. To simplify the calculations, consider the case when the excitations of alternators No. 1 and No. 2 are adjusted to make $E'_1 = E'_2$ and the governors so set that E'_1 and E'_2 are in phase. If under these conditions the rotor of No. 1 slips forward from its normal position an angle α the vector E'_1 takes up the position (E'_1) shown dotted in Fig. 80.

The change in current output of alternator No. 1 due to the displacement of E'_1 is

$$\underset{\cdot}{I}_c = \frac{(\underset{\cdot}{E}'_1) - E'_2}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2} \tag{17}$$

Since alternator No. 2 is assumed of much larger capacity than alternator No. 1 it follows that

$$\underset{\cdot}{I}_c = \frac{(\underset{\cdot}{E}'_1) - E'_2}{\underset{\cdot}{Z}_1} \quad \text{(approximately)} \tag{18}$$

The change in internal power developed in alternator No. 1 is (see Fig. 80).

$$\Delta P'_1 = q(E'_1)(AE) = qE'_1(AE) \quad \text{(approximately)} \tag{19}$$

Where q is the number of phases.

The change in power output of alternator No. 1 measured at the terminals is

$$\Delta P_1 = qV_1(AC) \tag{20}$$

If α is small, the drop

$$Z_1 I_c = E'_1 \alpha \quad \text{(approximately)} \tag{21}$$

If X_1 is large compared to r_1 which is usually true, the current

$$I_c = (AE) \quad \text{(approximately)}$$

Substituting this value in (21), we obtain

$$AE = \frac{E'_1 \alpha}{Z_1} \quad \text{(approximately)} \tag{22}$$

The total change in internal electrical power developed in alternator No. 1 is therefore

$$\Delta P'_1 = q\frac{(E'_1)^2 \alpha}{Z_1} \quad \text{(approximately)} \tag{23}$$

Let F = tangential force on rotor due to the change in internal power developed expressed in pounds.
r = radius of rotor in feet.
n = revolutions of rotor per second $\left(n = \frac{2f}{p_1}\right)$.

In terms of these quantities the change in internal power may be expressed by the relations

$$\Delta P'_1 = \frac{F2\pi rn}{550}746$$

or

$$\Delta P'_1 = \frac{Fr(2\pi)\left(\frac{2f}{p_1}\right)746}{550} \tag{24}$$

and from (23) and (24) the torque acting on the rotor is

$$T = Fr = \frac{550}{746}\,\frac{q(E'_1)^2\alpha p_1}{4\pi f Z_1} \quad \text{(approximately)} \tag{25}$$

The equation of motion of the rotor with respect to its equilibrium position is

$$\left(\frac{J_1}{32.2}\right)\frac{d^2\alpha'}{dt^2} = \text{torque in direction of } \alpha' \tag{26}$$

where J_1 is the moment of inertia of the rotor in pound foot units and α' is the displacement of the rotor in mechanical radians. Since $\alpha' = 2\frac{\alpha}{p_1}$ (where α is expressed in electrical space radians) it follows that equation (26) may be expressed by the relation

$$\frac{2}{p_1}\left(\frac{J_1}{32.2}\right)\frac{d^2\alpha}{dt^2} = \text{torque in the direction of } \alpha \tag{27}$$

where $\frac{d^2\alpha}{dt^2}$ = angular acceleration of rotor in electrical space radians per second per second when its displacement is α.

The torque is actually in the opposite direction to α and is given by equation (25), whence the equation of motion of the rotor about its equilibrium position is

$$\frac{d^2\alpha}{dt^2} = -\frac{(32.2)550qp_1{}^2(E'_1)^2\alpha}{(746)4\pi f(2)Z_1J_1} \tag{28}$$

or, putting

$$K_1 = \frac{(32.2)550qp_1{}^2(E'_1)^2}{(746)4\pi f(2)}$$

this equation becomes

$$\frac{d^2\alpha}{dt^2} = -\frac{K_1\alpha}{Z_1J_1} \tag{29}$$

This is the equation of harmonic motion, and the period of the oscillation, that is the natural period of the rotor, is

$$\text{Period} = 2\pi\sqrt{\frac{Z_1 J_1}{K_1}} \quad \text{(approximately)} \qquad (30)$$

The natural period of the rotor is therefore proportional to the square root of the synchronous impedance of the armature and also proportional to the square root of the moment of inertia of the rotor.

Therefore, the free period of the rotor may be increased either by increasing its moment of inertia (by adding a flywheel) or by inserting reactance in the armature circuit. The former method is the one usually employed, since inserting too much reactance reduces the stability of the rotor as explained later.

106. Synchronizing Power; Importance of Proper Synchronous Reactance.—The torque given by equation (25) is the difference between the electromagnetic torque due to the pull between the field and armature, and the driving torque of the prime mover. This torque is the torque which tends to pull the rotor into step when it slips an angle α from its normal position, and may therefore be called the " synchronizing torque." The greater the value of this synchronizing torque for a given value of α the greater will be the tendency of the rotor to remain in step.

Equation (25) shows that the synchronizing torque varies inversely as the synchronous impedance. It must be kept in mind that this equation holds only when the ratio of $\frac{X_1}{r_1}$ is very large. If X_1 is zero, the current vector I_a in Fig. 80 coincides with the line AB, and the component of I_a in phase with (E'_1) is practically zero, and the electromagnetic power $qE'_1(AE)$ is practically zero. A very small synchronous reactance or a very large synchronous reactance both produce unstable operation. The objection to increasing the free period of the rotor by increasing the armature reactance beyond a certain value is apparent.

107. Effect of Wave Form on Parallel Operation.—If the e.m.f. wave of one alternator contains a harmonic not in the e.m.f. of the other the loop formed by the two generators and the bus-bars will form a short-circuit to this harmonic, and a relatively large current of this frequency may be set up, causing useless heating and thereby limiting the output of the generators.

108. Effects of Speed—Load Characteristics of Prime Movers.—Since alternators which operate in parallel must always run at the same relative speed it follows that if they are to divide the load in the same proportion for all loads from no-load to full-load they must be driven by prime movers which have like speed load characteristics. That is, when the speed of each prime mover is plotted against its load expressed as a per cent of its full load these percentage characteristics for the several prime movers should coincide.

The percentage speed-load characteristics of prime movers are seldom exactly alike, but the effect of slight variations will be reduced if these characteristics have an appreciable droop. The speed-load characteristics of prime movers which drive alternators intended for parallel operation should usually droop about 3 per cent from no-load to full-load.

The effect of a periodic variation in the angular velocity during a revolution has already been discussed. A variation greater than one and one-quarter electrical space degrees is liable to render parallel operation unsatisfactory.

109. Sensitiveness of Governors.—The governors of the prime movers should not be too sensitive, i.e., they should not respond to momentary changes in load. The operation of the governors must be sluggish enough not to overrun the normal position corresponding to any change in load.

110. Summary of Conditions for Satisfactory Parallel Operation.—The alternators should have

1. The same voltage rating.
2. The same frequency.
3. The same wave form.
4. The synchronous impedances proportioned inversely as the ratings of the alternators.
5. The ratios of the synchronous reactances to the resistances in corresponding phases equal.
6. Prime movers operating at uniform angular velocities during each cycle, or a variation not exceeding one and one-quarter electrical space degrees.
7. Rotors whose free periods as torsional pendulums do not coincide with any periodic pulsation in the driving torque of the prime movers.
8. Prime movers whose speed-load characteristics from no-load to maximum-load are similar.

9. Prime movers of drooping speed-load characteristics.
10. Prime movers whose governors are not too sensitive.

PROBLEMS

1. Two alternators C and D, are operated in parallel. (a) What factors determine the power output of C? (b) How are the current outputs of the machines brought in phase with the load current? (c) Under what conditions are the current outputs of the alternators inversely proportional to the synchronous impedances of the armatures? (d) What adjustments are necessary to make the currents circulating among the machines zero at no-load?

2. Show that the power factor of an alternator operating in parallel with others becomes leading when its field excitation is lowered beyond a certain point.

3. The governor on the prime mover of an alternator operating in parallel with others of much larger capacity is set at a definite point. Why is it true that the output of this alternator cannot be changed (neglecting the change in the copper loss in its armature) by changing the field current?

4. Two identical 6500-kv-a., 12,000-volt, 25-cycle, three-phase, Y-connected alternators are operated in parallel and supply 10,000-kv-a. at a terminal voltage of 12,000 volts to a 95 per cent lagging power factor load. The following things are known about the alternators:

NO-LOAD SATURATION CURVE AT 25 CYCLES

Terminal Voltage	Field Current
2,000	30
4,000	60
6,000	90
8,000	120
10,000	160
12,000	210
14,000	270
16,000	360

Effective resistance per Y-phase at 75° C. = 1.73 ohms.

ZERO POWER FACTOR CURVE REFERRED TO 75° C.

(Line Current, 313 Amperes; Frequency, 25 Cycles)

Terminal Voltage	Field Current
0	100
2,000	125
4,000	160
6,000	190
8,000	230
10,000	280
12,000	340
14,000	425

One of the alternators operates at a field current of 310 amperes and delivers 4200 kilowatts. Calculate (*a*) the field current of the other alternator required to maintain a terminal voltage of 12,000 volts, at 25 cycles; and (*b*) the power factor at which each alternator operates. (*c*) Draw a complete vector diagram showing all the current and voltage relations. (*d*) What re-adjustments are necessary to make the alternators share the loads equally?

5. The alternators specified in Problem 4 are operated at no-load (no external load connected to the bus-bars). The voltage between bus-bars is 12,000 volts and the frequency is 25 cycles. The excitation voltage to neutral in a phase of one of the machines is 7500 volts and leads the bus-bar voltage to neutral in this phase by 12 time degrees. (*a*) Calculate the circulating current. (*b*) Calculate the electrical power which is transferred from one of the machines to the other.

6. Two 2300-volt, three-phase, Y-connected alternators are operated in parallel and supply an 850 kv-a. load at 2300 volts and 85 per cent leading power factor. The ratings and the constants of the machines are as follows:

Rating	Y-phase Effective Resistance	Y-phase Synchronous Impedance
750 kv-a.	0.141 ohm	2.31 ohms
500 kv-a.	0.220 ohm	2.50 ohms

The governors of the prime movers and the field excitations of the generators are so adjusted that each machine operates at the same power factor and delivers a current in direct proportion to its kv-a. rating. (*a*) Draw a complete vector diagram showing the voltage and current relations. (*b*) Calculate the excitation e.m.f.'s required and the phase angle between them.

CHAPTER VIII

SYNCHRONOUS MOTOR

A synchronous motor is an a-c. motor in which the normal speed of the rotor is strictly proportional to the frequency of the a-c. power supply. The relation between speed, frequency and number of poles is the same as that for a synchronous generator.

The construction of a synchronous motor is very similar to that of a synchronous generator, in fact any synchronous generator may be operated as a motor. However, for most satisfactory operation, a synchronous motor must be provided with salient poles and its pole faces must be provided with a substantial pole-face winding.

The chief advantages of a synchronous motor are that it may be used for improving the power factor of a reactive load or for controlling the voltage at the receiving end of a long transmission line. Its chief disadvantages are that the starting torque is small and that the rotor is subject to oscillation.

111. Principle of Operation of the Synchronous Motor.—(*a*) In either a single-phase or polyphase synchronous motor the directions of the currents in the armature coils must be such that during the major portion of a cycle the forces between the rotor and the stator produce a unidirectional torque. Thus, in Fig. 81, the current in the armature coil must reverse during the time required for the *N*-pole to move from the position marked 1 to the position marked 2. In other words, the current must complete ½ cycle in the time required for the main field to revolve through π electrical space radians and therefore the relation between speed and frequency must be definitely fixed.

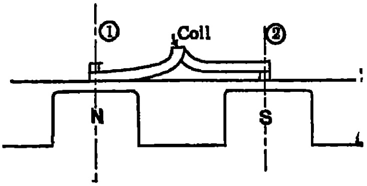


FIG. 81.—Synchronous Motor Showing One Armature Coil

(*b*) The armature winding on a polyphase synchronous motor is similar in every way to that on a polyphase synchronous generator. If polyphase currents are set up in the armature winding a revolving magnetic field is produced. The speed of this field (synchronous speed) is $2\frac{f}{p}$ revolutions per second, where f represents the frequency of the alternating-current supplied to the motor, and p the number of poles for which the armature is wound. To obtain an idea of how the revolving field is produced, the student should review the subject of armature reaction in alternators (see Article 83).

The electromagnetic reaction between the armature and the rotor fields depends on the relative velocities of the fields. Let us consider the reactions between the two fields in Fig. 82 when the rotor is at standstill.

When the south pole of the armature field approaches the north pole of the rotor field a clockwise impulse is given to the rotor which is balanced by an equal counter-clockwise impulse when the armature south pole leaves the rotor north pole. No motion of the rotor can take place, for the time of the impulses is very short and the inertia of the rotor is large. The conclusion may be drawn that a synchronous motor with no special attachments has practically no starting torque. (The hysteresis and eddy-current losses in the rotor iron of a polyphase motor produce a small torque.)

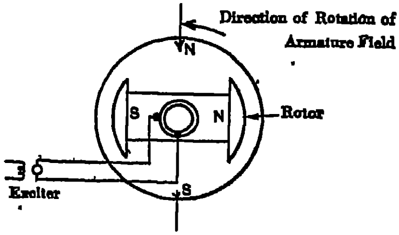


FIG. 82.—Diagram Illustrating Reaction Between Magnetic Fields in a Synchronous Motor

If the rotor is running near synchronous speed of the armature field the counter-clockwise impulses are of sufficient duration to accelerate the rotor to synchronism. At synchronous speed the forces acting on the rotor produce a unidirectional torque, and for a given mechanical load and rotor excitation the m.m.f's due to the armature currents alone and the m m.f's due to the field current alone remain a fixed distance apart.

CHAPTER IX

THEORY OF OPERATION OF THE SYNCHRONOUS MOTOR

112. Voltage Drop Current Relations for a Synchronous Motor.—In order to set up a current in a phase of an a-c. motor it is necessary that the impressed voltage be equal to the vector sum of the counter voltage drop due to the variation in the flux linking the phase and the voltage drop due to the resistance of the winding. This voltage may be expressed by any of the following relations:

$$\underset{\cdot}{V}_1=\underset{\cdot}{E}'''_1+r'_1\underset{\cdot}{I}_1 \tag{1}$$

$$\underset{\cdot}{V}_1=\underset{\cdot}{E}''_1+(r_1+jx_1)\underset{\cdot}{I}_1 \tag{2}$$

$$\underset{\cdot}{V}_1=\underset{\cdot}{E}'_1+(r_1+jX_1)\underset{\cdot}{I}_1 \tag{3}$$

where

V_1 = impressed phase voltage;
E'''_1 = voltage due to resultant flux linking a phase;
E''_1 = the voltage drop due to the flux produced by the field current and the armature current combined;
E'_1 = the voltage due to the flux produced by the main field current acting alone (the excitation voltage).
r'_1 = ohmic phase resistance;
r_1 = effective phase resistance;
x_1 = phase leakage reactance;
X_1 = phase synchronous reactance (this reactance corresponds to the synchronous reactance of a generator); and
I_1 = phase armature current.

Note that the difference between r'_1I_1 and r_1I_1 represents a small voltage and for this reason equation (1) may be written in the following form:

$$\underset{\cdot}{V}_1=\underset{\cdot}{E}'''_1+r_1\underset{\cdot}{I}_1 \quad \text{(approximately)}$$

The voltage relations may be represented by a diagram (see Fig. 83). This diagram is drawn for an excitation which results in a lagging power factor.

113. Power Entering and Leaving an Armature Phase of a Synchronous Motor.—The power entering an armature phase of a synchronous motor may be expressed by the product $V_1I_1 \cos \theta$. Naturally the difference between the power input and the copper loss ($V_1I_1 \cos \theta - r_1I_1^2$) represents the power which leaves each phase of the armature winding. The major portion of this power is transferred across the air-gap to the rotor and the rest is lost in the stator iron. The power leaving a phase of the stator may be expressed in terms of the electromotive forces as follows:

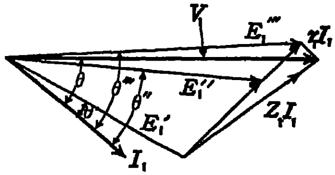


Fig. 83.—Vector Diagram for a Phase of a Synchronous Motor

$$P_M = V_1I_1 \cos \theta - r_1I_1^2 = E'''_1I_1 \cos \theta''' = E''_1I_1 \cos \theta''$$
$$= E'_1I_1 \cos \theta' \qquad (4)$$

The quantity P_M is called the phase motor power. The mechanical power output of q phases of the motor is

$$P = qP_M - \text{Iron loss} - \text{Friction and windage} \qquad (5)$$

114. Diagram Showing Variations of Excitation Voltages and Power Factors for Different Loads on a Synchronous Motor.—The following diagrams are approximate in that the phase angle of the synchronous impedance β is assumed constant. In each diagram the voltage impressed on the motor is also assumed constant.

Variation of excitation voltage and power factor for constant power input (Fig. 84).—In this diagram β is assumed equal to 90 degrees; i.e., $r_1I_1^2 = 0$. For a given power input $I_1 \cos \theta$ is constant and $X_1I_1 \cos \theta$ is approximately constant. In other words,

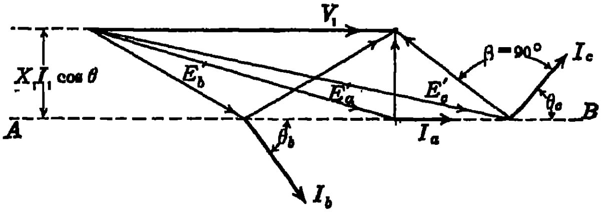


Fig. 84.—Diagram Showing Variation of the Power Factor of a Synchronous Motor when the Excitation E. M. F. is Changed

for a given input the line AB is parallel to V_1. This diagram shows that for a given impressed voltage and a given load the current and the power factor are determined by the excitation voltage.

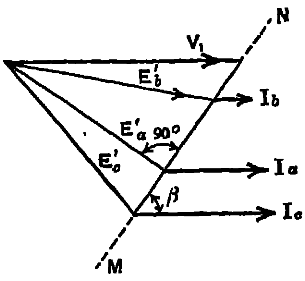


Fig. 85.—Diagram Showing Variation of the Excitation E. M. F. of a Synchronous Motor when the Load is Changed and the Power Factor Maintained at Unity

Variation of excitation voltage required to maintain unity power factor for different power inputs.—In Fig. 85 the impedance drop line MN makes a constant angle with V_1.

The diagram shows that the excitation voltage required to maintain unity power factor for different loads is not constant; a minimum excitation $E'a$ is required when the phase input is V_1I_a. If the armature had zero resistance, the line MN would be perpendicular to V_1 and the excitation voltage would be a minimum when the input is a minimum.

Variation of power factor with load at constant excitation voltage (Fig. 86).—The phase relations of the excitation e.m.f. and the impedance drop to the impressed voltage depend on the mechanical load on the motor. For this reason the power factor of the motor depends on the mechanical load.

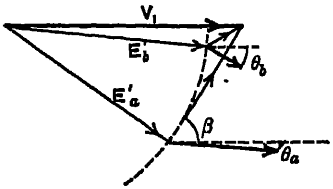


Fig. 86.—Diagram Showing Variation of Power Factor of a Synchronous Motor when the Excitation E. M. F. is Held Constant and the Load on the Motor Changed

Summary (see Figs. 84, 85, 86).—

1. The power factor of a synchronous motor depends on the excitation voltage and the load on the motor.

2. The excitation voltage may be increased to a point where the motor takes a leading current from the supply.

3. The excitation required to maintain constant power factor for different loads is not constant, but depends on the load and the constants of the motor.

115. Experimental Determination of V-curves and Compounding Curves.—A V-curve (so-called on account of its shape) is a curve showing the relation between the armature currents and excitation voltages of a synchronous motor operating at constant impressed voltage, frequency, and power input. For a given input the armature current is a minimum when the excitation is so adjusted that unity power factor is obtained. Excitation voltages above or below this value result in armature currents which are greater than those obtained for unity power factor.

Experimentally a V-curve may be obtained as follows: The power input to the motor is held constant by adjusting the mechanical load on the motor; the impressed voltage and frequency are kept constant by adjusting the speed and the field current of the alternator supplying the motor; and measurements are made of the motor armature currents for different motor field currents. The actual values of the excitation voltages corresponding to the different field currents may be obtained from the no-load saturation curve of the motor.

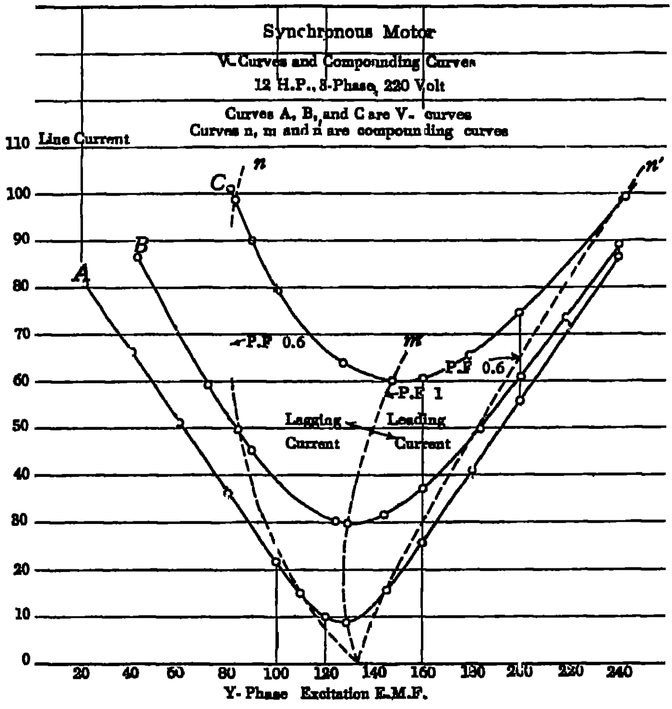


FIG. 87.—V-curves and Compounding Curves for a Synchronous Motor

A curve showing the relation between the armature currents and excitation voltages to maintain constant power factor for different loads is called a compounding curve. A family of V- and compounding curves are shown in Fig. 87.

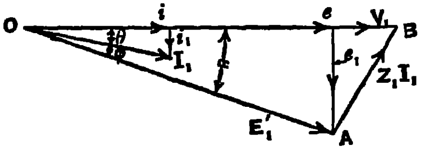


Fig. 88.—Vector Diagram for a Synchronous Motor Showing Components of Excitation Voltage and Current

116. Mathematical Analysis of the Performance of a Synchronous Motor.—The motor power of a synchronous motor may be expressed in terms of the power factor angle θ and the electrical space displacement α of the rotor. Thus referring to Fig. 88 the motor power may be expressed by the relation

$$P_M = E'_1 I_1 \cos(\theta-\alpha) \tag{6}$$

$$= E'_1 I_1 \cos\theta\cos\alpha + E'_1 I_1 \sin\theta \sin\alpha$$

$$= ei + e_1 i_1 \tag{7}$$

where e, e_1 and i, i_1 are the components of the excitation voltages and the armature current respectively.

The components of the current referred to the impressed voltage may be calculated by the relation

$$\underset{\cdot}{I}_1 = \frac{\underset{\cdot}{V}_1 - \underset{\cdot}{E}'_1}{\underset{\cdot}{Z}_1} = i - ji_1 = \frac{v-e+je_1}{\underset{\cdot}{Z}_1}$$

$$= \frac{[r_1(v-e)+X_1e_1] - j[X_1(v-e)-r_1e_1]}{r_1^2+X_1^2} \tag{8}$$

and

$$P_M = \frac{[r_1(v-e)+X_1e_1]e + [X(v-e)-r_1e_1]e_1}{r_1^2+X_1^2}$$

Since $e = E'_1 \cos\alpha$; $e_1 = E'_1 \sin\alpha$; and $V_1 = v$ it follows that

$$P_M = \frac{V_1E'_1[r_1\cos\alpha + X_1 \sin\alpha] - r_1E'^2_1}{r_1^2+X_1^2} \tag{9}$$

For a given value of E'_1, V_1, r_1, and Z_1 the maximum motor power occurs when $\frac{dP_M}{d\alpha}=0$. The approximate value of α under these conditions may be determined by differentiating equation (9) and equating the result to zero. Thus

$$\frac{dP_M}{d\alpha} = \frac{V_1E'_1}{Z_1^2}[-r_1\sin\alpha + X_1\cos\alpha] = 0$$

and

$$\alpha = \tan^{-1}\frac{X_1}{r_1} = \beta \qquad (10)$$

Substituting for sin α the value $\frac{X_1}{Z_1}$ and for cos α the value $\frac{r_1}{Z_1}$ in equation (9), we obtain for the maximum motor power at a given excitation the expression

$$P_{MM} = \frac{V_1 E'_1}{Z_1} - \frac{r_1(E'_1)^2}{Z_1^2} \qquad (11)$$

The excitation voltage required to obtain the maximum possible motor power may be obtained by differentiating equation (11) with respect to E'_1 and equating the result to zero. Thus

$$\frac{dP_{MM}}{dE'_1} = \frac{V_1}{Z_1} - \frac{2r_1 E'_1}{Z_1^2} = 0 \qquad (12)$$

and

$$E'_1 = \frac{Z_1 V_1}{2r_1}$$

The expression for the maximum possible theoretical motor power is obtained by substituting this value of E'_1 in equation (11). By carrying out this substitution, we obtain

$$P'_{MM} = \frac{V_1^2}{4r_1} \text{ watts per phase} \qquad (13)$$

By making $E'_1 = \frac{Z_1 V_1}{2r_1}$ and $\alpha = \beta$ in Fig. 88 it is readily shown that the maximum possible motor power occurs when the impedance drop

$$Z_1 I_1 = \frac{Z_1 V_1}{2r_1} = E'_1$$

and that the current under these conditions

$$I_1 = \frac{V_1}{2r_1} \qquad (14)$$

The power factor of the motor when the maximum possible motor power occurs may be calculated by substituting (13) and (14) in the relation

$$P_M = V_1 I_1 \cos\theta - rI_1^2$$

Thus

$$\frac{V_1^2}{4r_1} = V_1\left(\frac{V_1}{2r_1}\right)\cos\theta - r_1\left(\frac{V_1^2}{4r_1^2}\right)$$

or

$$\cos\theta = 1 \qquad (15)$$

Conclusions based on equations (10–15).—

1. Equation (10) shows that for a given V_1, E'_1, r_1 and Z_1 the maximum motor power or approximately the maximum torque occurs when the excitation voltage of the motor is displaced $\alpha = \tan^{-1}\frac{X_1}{r_1}$ electrical time degrees with reference to the corresponding phase voltage of the alternator supplying the motor.

2. Equation (11) shows that for a given E'_1, r_1, and Z_1 the maximum motor power of a synchronous motor is some function of the first power of the impressed voltage.

3. Equation (11) also shows that for a given r_1, Z_1, and $E'_1 =$ constant times V_1, the maximum motor power is a function of the square of the impressed voltage.

4. Equation (15) shows that the maximum theoretical motor power is obtained when the power factor of the motor is unity.

5. Since the resistance of an armature winding is very small compared with the numerical value of the impressed voltage, it follows that the armature current at which the maximum possible power occurs is far beyond the current rating of the machine.

117. Current Circle Diagram for a Synchronous Motor.— The performance curves of a synchronous motor may be obtained from a current circle diagram. The diagram used is based on the assumption that V_1, r_1, and Z_1 are constant throughout the entire operating range of the motor.

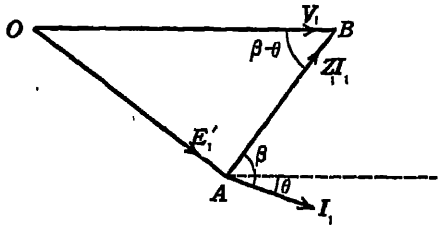


FIG. 89.—Vector Diagram for a Synchronous Motor

The current circle diagram is obtained from the voltage drop-current diagram in Fig. 89, by dividing V_1, E'_1, and Z_1I_1 by Z_1. Thus in Fig. 90, which represents the circle diagram, the line $OP = \frac{V_1}{Z_1}$ represents the current in the armature when the excitation voltage is zero; the distance $PS = \frac{E'_1}{Z_1}$; and the distance

$$OS = \frac{Z_1I_1}{Z_1} = I_1$$

The triangle *PSO* (Fig. 90) is similar to the triangle *OAB* (Fig. 89). The angle POS = angle $OBA = \beta - \theta$. The angle $QOP = \beta$ and therefore the angle $QOS = \theta$, the power factor angle of the motor.

The construction of the circle diagram is completed by bisecting the line *OP* and erecting the perpendicular *LQ*. The distance *OQ* equals the distance *PQ*; the length of each of these lines is

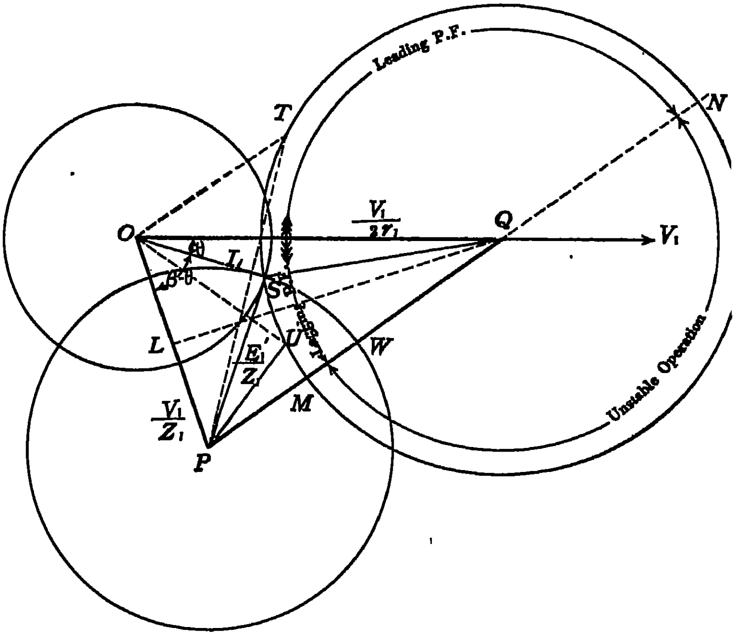


FIG. 90.—Current Circle Diagram for a Synchronous Motor

$$\frac{OL}{\cos\beta} = \frac{\dfrac{V_1}{2Z_1}}{\dfrac{r_1}{Z_1}} = \frac{V_1}{2r_1}$$

It follows that the line *OQ* represents the armature current for the maximum possible theoretical power [see equation (14)] and the line $PQ(Z_1)$ is the excitation voltage required to obtain the maximum possible motor power.

The line SQ is a function of the motor power and may be calculated by the following method:

$$\overline{SQ}^2=\left(\frac{V_1}{2r_1}\right)^2+I_1^2-2\left(\frac{V_1}{2r_1}\right)I_1\cos\theta_1$$

$$\overline{SQ}^2=\left(\frac{V_1}{2r_1}\right)^2+\frac{r_1I_1^2-V_1I_1\cos\theta_1}{r_1}$$

$$\overline{SQ}^2=\frac{V_1^2}{4r_1^2}-\frac{P_M}{r_1}$$

therefore

$$SQ=\frac{V_1}{2r_1}\sqrt{1-\frac{4r_1P_M}{V_1^2}} \tag{16}$$

When the motor power has the maximum possible value, SQ is zero and

$$\sqrt{1-\frac{4r_1P'_{MM}}{V_1^2}}=0$$

therefore

$$P'_{MM}=\frac{V_1^2}{4r_1} \tag{17}$$

Conclusions based on the circle diagram:

1. For a given V_1, r_1, and Z_1, and for an excitation voltage $(PS)\times Z_1$, the maximum motor power is a function of the distance WQ.

2. For a given motor power SQ the minimum theoretical excitation voltage is $(PM)\times Z_1$ and the maximum theoretical excitation voltage is $(PN)\times Z_1$.

3. For a given motor power SQ the theoretical operating range is greater for leading than for lagging power factor.

4. If $OT=OU=$rated armature current for the motor and SQ is constant. The angle TOQ is the maximum angle of leading current and the angle UOQ the maximum angle of lagging current at which the machine may be continuously operated. It is assumed that the current rating of the armature is not influenced by changes in the field current.

118. Starting Torque of Synchronous Motors.—The armature current in a single-phase synchronous motor produces an alternating field which is fixed in position relative to the armature conductors. This field produces absolutely no starting torque and for this reason all single-phase synchronous motors must be synchronized by one of the methods outlined in the chapter on parallel operation of alternators. Single-phase synchronous motors on account of their unstability of operation are seldom used and further consideration of their operation is not required.

In a polyphase motor the field produced by the armature currents is nearly constant in magnitude and revolves with reference to the armature conductors. This field cuts across the main field poles (the rotor) and a unidirectional torque is exerted on the rotor. The starting torque is due to the following things:

1. Hysteresis in the rotor iron.
2. Eddy-currents in the rotor iron.
3. Eddy-currents in the pole-face winding.

In the discussion which immediately follows it is assumed that the rotor winding (the main field winding) is open-circuited.

The torque due to the hysteresis of the rotor iron is caused by the lag of the resultant rotor magnetization behind the axis of the revolving magnetomotive force of the stator winding. That is, magnetic hysteresis causes an angular displacement (see Fig. 91) between the poles formed on the stator and the rotor and as a result of this displacement a small torque is exerted on the rotor. The direction of the torque is the same as that of the revolving magnetomotive force.

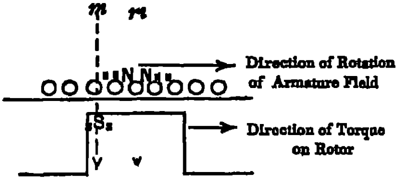

m
n
N
N
S
Y
Direction of Rotation of Armature Field
Direction of Torque on Rotor

FIG. 91.—Starting Torque of an A-c. Motor Due to Magnetic Hysteresis

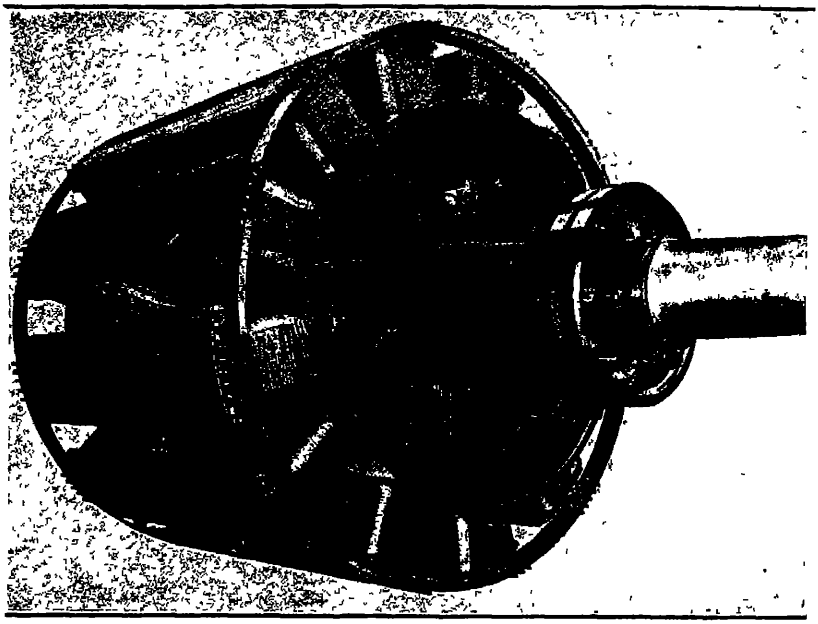
The torque due to the eddy-current losses in the rotor iron is small compared with that produced by the currents induced in the pole-face winding. The pole-face winding shown in Fig. 92 consists of copper bars embedded in the pole faces and short-circuited by end rings. Any relative motion between these bars and the air-gap field results in a large circulating current through them. The field produced by these currents reacts on the armature field and a torque is developed tending to rotate the rotor in the same direction as the rotation of the armature field. The magnitude of this torque depends on the following things:

1. Resistance and reactance of pole-face windings.
2. Relative speed between the armature field and the rotor.

3. Magnitude of armature field.
4. Radius of rotor.

(See induction motor theory.)

By proper design of the pole-face winding it is possible to construct a synchronous motor having a high starting torque and a high pull-in (synchronizing) torque. A high torque near synchronous speed is essential to obtain exact synchronism. In order to understand how exact synchronism is obtained the reader should review part (*b*), " Principle of Operation of the Synchronous Motor," Chapter VIII.



(By the courtesy of the Westinghouse Co.)

FIG. 92.—Rotor of a Synchronous Motor Showing Pole-face Winding

119. Induction Motor Method of Starting a Polyphase Synchronous Motor.—Any properly designed polyphase synchronous motor may be started like an ordinary squirrel-cage induction motor. The method consists in connecting the stator winding of the motor to the supply through a starting compensator (auto-transformer). The initial value of the voltage applied to the winding is approximately 60 per cent of the rated voltage of the motor. During the starting period the armature field cuts across the main field coils and to prevent a high voltage from being induced in them it is necessary to either short-circuit the field winding or to supply it with a small direct-current excitation. When synchronous speed is reached (one can tell by the sound of the machine and the magnitude of the armature current) the field current is adjusted until the desired power factor is obtained.

120. Hunting.—Hunting in synchronous motors is similar to that produced when alternators are operated in parallel. A periodic change in the mechanical load, or a sudden change in the mechanical load or supply frequency results in rotor oscillations. The amplitude of these oscillations is greatly reduced by the pole-face winding (armortisseur windings). When the rotor oscillates a relative motion between the air-gap flux and the pole-face winding takes place, currents are induced in the pole-face winding and forces are set up in opposition to those producing the oscillations. A pole-face winding on a synchronous motor serves two useful purposes, namely: the starting torque and the stability of operation are increased.

121. Power Factor Correction with Static and Synchronous Condensers.—Power is most economically generated and distributed at unity power factor. By the use of suitable equipment it is always possible to obtain unity power factor or very close to it. A consumer of power operating at a low-power factor is required to pay higher rates than one operating at a high-power factor, and for this reason power factor correcting devices should be connected in parallel with inductive loads. The power factor of a reactive load may be improved by connecting in parallel with the load either a static or a synchronous condenser, depending on the magnitude of the load and its power factor.

Power factor correction with static condensers (parallel plate condensers) is limited to relatively small inductive loads. On low-voltage circuits the static condenser is connected across the secondary winding of a step-up transformer and the primary winding of the transformer is connected in parallel with the inductive load. The power factor of the primary winding of the transformer is approximately equal to that of the secondary winding and for this reason a transformer and condenser combined enable one to obtain a large condensive load by the use of a relatively small condenser. On high-voltage circuits the condenser is sometimes connected directly to the line. If the voltage is too high it may be necessary to connect the condenser to the secondary winding of a step-down transformer.

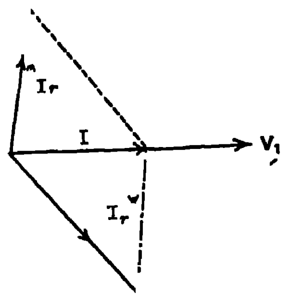
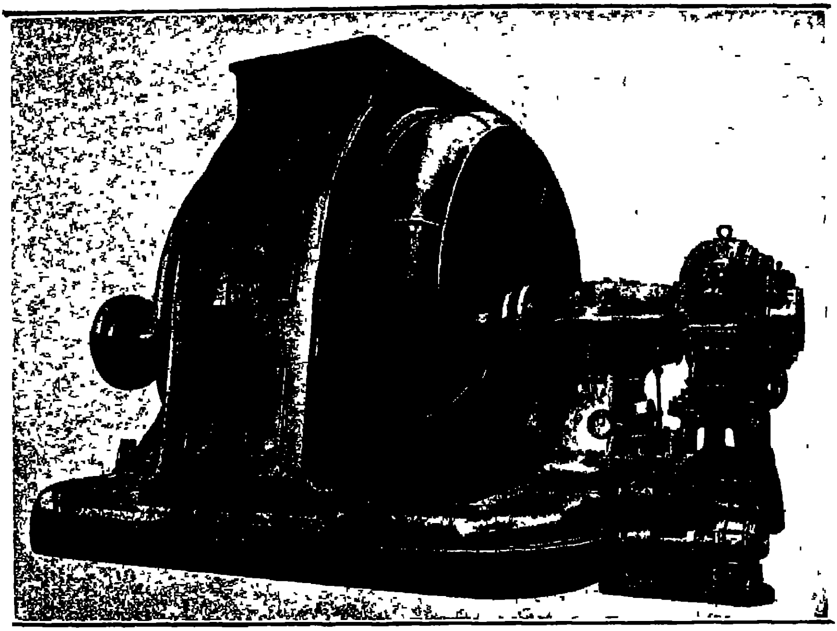


FIG. 93.—Power Factor Correction by Condensers

The power factor of large reactive loads is readily corrected by a synchronous motor operating in parallel with the load. The reactive component of the current taken by a synchronous motor is determined by the field excitation of the motor and may be made to balance the reactive component of the load current. Thus, in Fig. 93, the reactive component I'_r of the motor current is made equal to the reactive component I_r of the load current and as a consequence the resultant current in the supply circuit is in phase with the load voltage.



(By the courtesy of the General Electric Co.)

FIG. 94.—Synchronous Condenser

A synchronous motor when used for balancing the reactive component of the current taken by an inductive load is called a "Synchronous Condenser." The construction of a synchronous condenser (Fig. 94) is similar to that of a synchronous motor. Due to the fact that the external mechanical load on the synchronous condenser is zero a much smaller shaft may be used than that required for an ordinary motor operating at the same voltage and current.

122. Transmission Line Voltage Control with a Synchronous Condenser.—The voltage at the receiving end of a long transmission line under certain conditions is subjected to wide fluctuations.

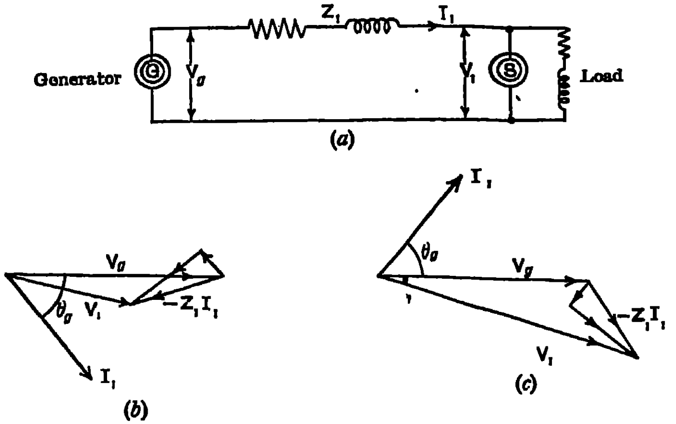


FIG. 95.—Voltage Control at the End of a Transmission Line by Using a Synchronous Condenser

At no-load a long line requires a considerable charging current and the drop in the line due to this current when subtracted vectorially from the generator voltage results in a much higher receiving end voltage than that furnished by the generator. On the other hand, if the transmission line supplies an inductive load the voltage drop in the line subtracted vectorially from the generator voltage results in a much lower load voltage than that across the terminals of the generator.

A nearly constant voltage may be obtained by connecting a synchronous condenser across the receiving end of the line. If for any reason the voltage across the synchronous condenser tends to increase above the normal value the synchronous machine takes a lagging current from the line and the line drop caused by the lagging current tends to prevent the increase in voltage. Similarly if the voltage across the synchronous condenser decreases below its normal value the machine takes a leading current from the line and the line drop now tends to increase the load voltage. Fig. 95*a* represents a simplified equivalent circuit for a transmission line; in this special case the electrostatic capacity of the line is neglected. Fig. 95*b* shows that a lagging current through this particular line tends to lower the load voltage and Fig. 95*c* shows that a leading current through the line tends to raise the load voltage.

PROBLEMS

1. (*a*) Show by a series of diagrams that when a two-phase current is set up in a two-phase armature winding a revolving magnetic field is produced. (*b*) Show that the speed of the revolving field is strictly proportional to the frequency of the current.

2. The following things are known about a 220-volt, 6-pole, 60-cycle, three-phase synchronous motor:

Effective resistance per delta-phase at the operating temperature, 0.109 ohm;

Synchronous impedance per delta-phase at the operating temperature, 0.84 ohm;

The motor is connected to a 220-volt, 60-cycle, balanced three-phase supply, and the current per terminal is 35 amperes. The field current of the motor is so adjusted that the excitation voltage is 220 volts. (*a*). Calculate: (1) the speed of the motor; (2) the power factor at which the motor operates; and (3) the total power which leaves the armature winding of the motor. (*b*) Construct a complete voltage drop-current vector diagram for the motor.

3. The following things are known about a 12-h.p., 220-volt, three-phase, 60-cycle, Y-connected synchronous motor:

No-load Saturation Curve

(Frequency, 60-Cycles)

Terminal Voltage	Field Current
146	4
199	6
238	8
264	10
284	12
296	14

ZERO POWER FACTOR CURVE

(Frequency, 60 Cycles; Temperature, 25° C., and Line Current, 39.4 Amperes)

Terminal Voltage	Field Current
0	6.9
158	12.0
199	14.0
229	16.0

D-c. resistance between terminals (measured at 25° C.) 0.253 ohm.

Effective resistance = 1.35 times d-c. resistance at 75° C.

Total friction + windage + core loss = 1.95 kw. (Assume that this loss is constant for all loads.)

The motor is connected to a balanced 220-volt, 60-cycle, three-phase supply and delivers 7.5 h.p. at the pulley. The power factor of the motor is unity and its operating temperature is 75° C. Calculate the field current required by the motor. (The motor is excited from a separate source.)

4. (*a*) Predetermine a V-curve for the motor specified in Problem 3 corresponding to 5 h.p. output at the pulley. (The excitation voltages may be calculated by analytical methods or may be obtained from the circle diagram.)

(*b*) What assumptions did you make in solving part (*a*)?

5. The synchronous motor specified in Problem 3 is operated from a balanced 220-volt, 60-cycle, three-phase supply. The motor delivers 4 h.p. and operates at unity power factor. Calculate: (*a*) the field current of the motor; and (*b*) the power factor of the motor when the frequency, field current, and the load are the same as in part (*a*) but the impressed voltage is 15 per cent greater.

6. The synchronous motor in Problem 3 is operated from the same power supply as a group of small induction motors. The total power input to the induction motors is 10.7 kv-a. at 74 per cent lagging power factor. The output of the synchronous motor is 6 h.p. The line voltage is 220 volts and the frequency 60 cycles. The field current of the synchronous motor is so adjusted that the combined load operates at unity power factor. (*a*) Calculate: (1) the current input to the induction motors; (2) the current input to the synchronous motor; (3) the field current of the synchronous motor; and (4) the current input to the combined load. (*b*) Draw a complete voltage drop-current vector diagram for this particular load.

7. The synchronous motor in Problem 3 is operated at rated output. (*a*) Calculate the maximum and minimum theoretical excitation voltages at which this motor will operate. (*b*) Do you consider these voltages practical? Why? (*c*) What assumptions did you make in the solution of this problem?

8. The synchronous motor in Problem 3 is operated from a 220-volt, 60-cycle balanced three-phase source. Calculate the maximum theoretical motor power possible.

9. The synchronous motor in Problem 3 is operated at the end of a three-phase transmission line. The motor is driven by a generator whose constants are equal to the constants of the motor (the machines are identical). The fields of the two machines are excited from entirely independent sources. The field current in each machine is 9 amperes. The frequency is 60 cycles. The transmission line has a resistance of 0.31 ohm and an inductive reactance of 0.4 ohm (each wire). The load on the synchronous motor is 4 h.p. The operating temperature of the machines is 25° C. (*a*) Calculate: (1) the voltage across the terminals of the motor; (2) the voltage across the terminals of the generator; and (3) the phase angle between the excitation e.m.f.'s of the two machines. (*b*) Construct a complete vector diagram showing the voltage and current relations.

10. A certain synchronous motor is operated at constant frequency and at constant field current. Prove that the maximum theoretical torque is a function of the first power of the impressed voltage.

11. A certain synchronous motor is operated at constant frequency and at constant power factor. Prove that the maximum theoretical torque under these conditions is a function of the square of the impressed voltage.

CHAPTER X

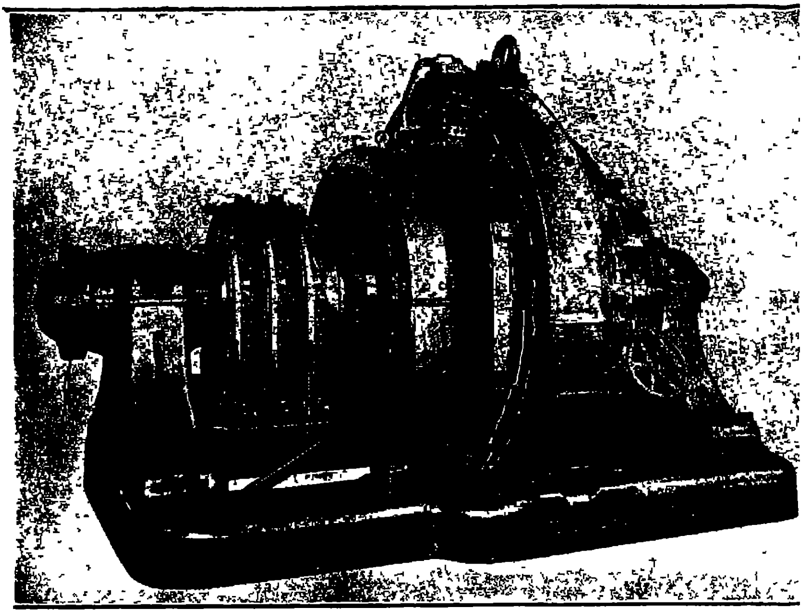
RECTIFICATION OF ALTERNATING-CURRENT

Many receiving circuits require direct-current and must be supplied by either a d-c. generator or through a rectifying device operated from an a-c. circuit. Small quantities of d-c. power may be obtained from a-c. sources through mercury-vapor, thermionic, electrolytic, or mechanical rectifiers. Street-railway systems and electro-chemical processes requiring large quantities of d-c. power are supplied by synchronous converters or by motor-generator sets.

123. Synchronous Converter.—A synchronous converter is a motor-generator in which a single armature winding serves the double purpose of a motor-armature winding and a generator-armature winding. If this type of machine is used to transform a-c. power to d-c. power its speed is strictly proportional to the frequency of the a-c. supply and the d-c. voltage obtained is proportional to the a-c. voltage of the supply.

The synchronous converter is the most efficient and most reliable revolving electrical machine. Its efficiency is close to that of a stationary transformer and is considerably higher than that of either the motor or the generator element of a motor-generator set having separate armatures. Commutation difficulties and flexibility of voltage control originally limiting the use of this type of rectifier have been entirely overcome. A modern synchronous converter known as a synchronous-booster converter is shown in Fig. 96. Two armatures are used in this particular machine. One of the armatures is relatively small and serves merely to boost or buck the a-c. voltage before it is impressed on the main armature winding. Fig. 97 illustrates how this particular converter is connected.

124. Armature Winding of a Synchronous Converter.—The armature winding of a synchronous converter is similar to that of a d-c. generator; a mesh connected winding is required. Elec-



(By the courtesy of the General Electric Co.)

FIG. 96.—Synchronous Converter

trical connections are made to a commutator at one end of the armature winding and to collector-rings at the other end. In a three-phase machine, three collector-rings are required and the

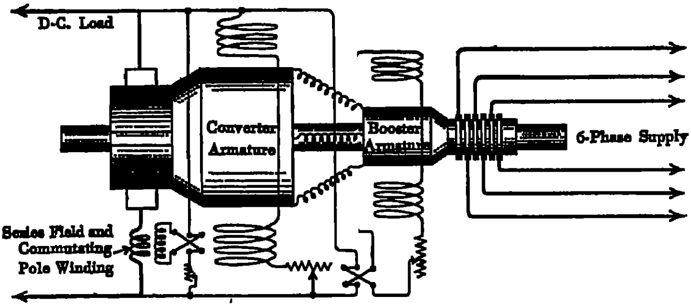


FIG. 97.—Wiring Diagram of a Synchronous Booster Converter

armature winding is tapped every 120 electrical space degrees. In general the number of tapping points required on a polyphase converter is equal to the product of the pairs of poles by the number of phases $\left(\frac{p}{2}q\right)$. The connections to the commutator are similar to those in a d-c. generator. Fig. 98 illustrates how the armature winding of a three-phase machine is connected to the collector-rings and to the commutator.

A given converter armature connected for six-phase operation has approximately two times the capacity of a like armature connected for single-phase operation. For this reason and the fact that power transformations from three-phase to six-phase are readily made, converters are usually connected for six-phase operation.

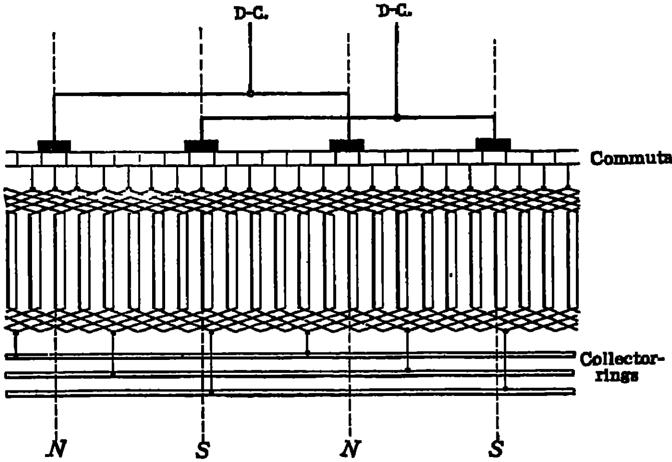


FIG. 98.—Armature Winding of a Synchronous Converter

125. Theoretical Voltage and Current Ratios of a Synchronous Converter.—In supplying a d-c. load by a synchronous converter one is interested in accomplishing the transformation from a-c. to d-c. efficiently and at a minimum outlay of capital. Since the a-c. transmission circuits and the converter circuits usually operate at decidedly different voltages it is seldom possible to make the transformation by direct connection of the converter to the source of power; i.e., transformers are necessary to form the proper connecting link between the supply and the converter. In order to determine the voltage and current ratings of the transformers it is necessary to know the relations between the voltages and the currents on the a-c. and d-c. sides of the converter.

Voltage ratio.—In order to determine the voltage ratio 180 electrical space degrees of the armature winding in Fig. 99 will be considered. On this diagram the armature winding taps to adjacent collector-rings are located $\frac{360}{q}$ electrical space degrees apart; the d-c. brushes are located 180 electrical space degrees apart; and the armature slots are δ electrical space degrees apart.

The armature coils cut across the same field flux and for this reason the r.m.s. values of the e.m.f.'s induced in them are equal. On account of the location of the coils the e.m.f.'s in adjacent coils are δ time degrees out of phase. This is so because in any synchronous machine the electrical time angle between the e.m.f.'s in the coils is equal to the electrical space angle between the physical locations of the coils. On this basis it follows that the vectors representing the voltages in the coils located between *NM* (180 electrical space degrees of the winding in Fig. 99) placed end to end as shown in Fig. 100, form a semi-circle.

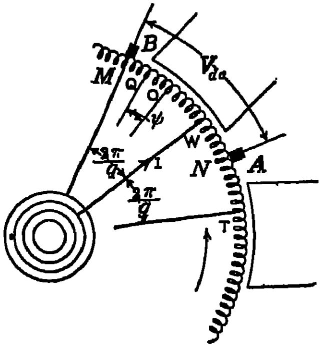


FIG. 99.—A Portion of the Armature of a Synchronous Converter

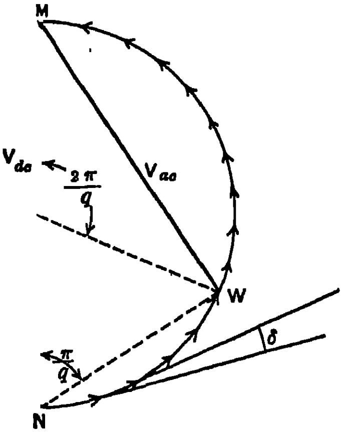


FIG. 100.—Vector Diagram of the Voltages in a Synchronous Converter

The d-c. voltage (V_{dc}) is equal to the sum of the instantaneous voltages induced in the coils located between *N* and *M*; it is equal to the length of the vector *NM*. The maximum value of the a-c. voltage induced between adjacent rings is equal to the vector sum of the voltages induced in the coils located between *W* and *M* and is equal to the vector WM. From Fig. 100 it follows that

$$V_{ac}(\text{max.}) = V_{dc}\sin\frac{\pi}{q}$$

or

$$V_{\text{ring}} = V_{ac}(\text{r.m.s.}) = \frac{V_{dc}}{\sqrt{2}}\sin\frac{\pi}{q} \qquad (1)$$

Equation (1) neglects the armature resistance and reactance drops and is an approximation; ordinarily the armature drop is small.

Current ratio.—The a-c. power input to the portion of the armature winding WM in Fig. 99 is $V_{\text{ring}}I_{\text{coil}}\cos\theta$, where I_{coil} represents the r.m.s. value of the a-c. component of the current in any of the coils located between WM. The total power input for 360 electrical space degrees of the armature winding is $qV_{\text{ring}}I_{\text{coil}}\cos\theta$ and the total input for the whole armature is $\frac{p}{2}qV_{\text{ring}}I_{\text{coil}}\cos\theta$. The total input multiplied by the efficiency (ξ) gives the total d-c. output. That is

$$\left(\frac{p}{2}\right)qV_{\text{ring}}I_{\text{coil}}\cos\theta\,\xi = V_{dc}I_{dc} \qquad (2)$$

This formula enables one to calculate the a-c. component of the coil current.

A formula showing the relation between the total alternating-current entering a ring and the total d-c. output may be obtained by substituting for I_{coil} its value in terms of I_{ring}.

The total current entering a collector-ring is

$$I_{\text{ring}} = \frac{p}{2}I_{\text{tap}} \qquad (3)$$

The tap current (I_{tap})·the current entering say tap No. 1 in Fig. 99, is equal to the vector sum of the coil currents I_{WM} and I_{WT}. The currents from M to W and from W to T are $\frac{2\pi}{q}$ time degrees out of phase and have equal r.m.s. values. The tap current is obtained as indicated by Fig. 101. The magnitude of this current is

$$I_{\text{tap}} = 2I_{\text{coil}}\sin\frac{\pi}{p} \qquad (4)$$

Substituting (1), (3), and (4) in equation (2), we obtain

$$I_{\text{ring}}=\frac{2\sqrt{2}\,I_{dc}}{q\cos\theta\xi} \tag{5}$$

This is also an approximation because the voltage drop in the winding is neglected.

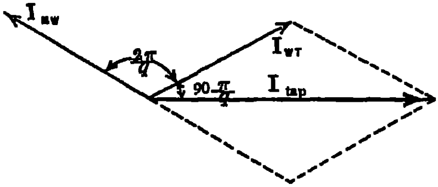


Fig. 101.—Vector Diagram Showing the Coil and Tap Current in a Synchronous Converter

126. Resultant Current in an Armature Coil of a Converter.—In a synchronous converter, motor and generator actions take place in the same armature winding. For this reason the actual current in any coil may be considered the resultant of two components. The motor current flows in opposition to the generated e.m.f., and the generator current flows in the direction of the e.m.f.

To obtain an expression for the actual current in any coil, such as coil Q in Fig. 99, it is necessary to determine the magnitudes and wave shapes of the a-c. and d-c. components.

Magnitude and wave form of d-c. component.—The magnitude of the d-c. component is determined by dividing the total output current by the paths in parallel. For a lap wound machine the d-c. component in any coil is

$$I_{dc}(\text{coil})=\frac{I_{dc}}{p}=\frac{I'_{dc}}{2} \tag{6}$$

where I'_{dc} is the direct-current in one brush set.

The wave shape of the d-c. component is rectangular, for the simple reason that the current in any armature coil of a d-c. generator is constant in magnitude and direction during the time required for the coil to move from one brush to the next. Thus, when coil Q is commutated by brush A, the current in it reverses and remains constant until the coil is commutated by brush B. The time at which a coil undergoes commutation depends on the location of the coil on the armature and for this reason the d-c. component waves are different for each coil located between two adjacent brush sets.

Magnitude and wave form of the a-c. component.—The r.m.s. value of the a-c. component of the coil current is obtained by solving equation (2). The magnitude of this component is

$$I_{ac}(\text{coil}) = \frac{V_{dc}I_{dc}}{\left(\frac{p}{2}\right)qV_{\text{ring}}\cos\theta\,\xi} \tag{7}$$

A sinusoidal wave shape is assumed. The phase relation of this current with respect to the phase impressed voltage depends on the power factor at which the machine is operated. The operation of a converter on the a-c. side is similar to that of a synchronous motor; i.e., the power factor is determined by the field current and the impressed voltage.

Wave shape of the resultant current in any coil located (ψ) electrical space degrees from the central coil of a phase.—The wave shapes of the resultant currents in all the coils constituting a given phase band are different. This is evident when we keep in mind that the a-c. components in the coils of a phase band (such as *WM* in Fig. 99) are equal and in phase, and that the d-c. components are equal, but do not reverse (commutate) at the same time.

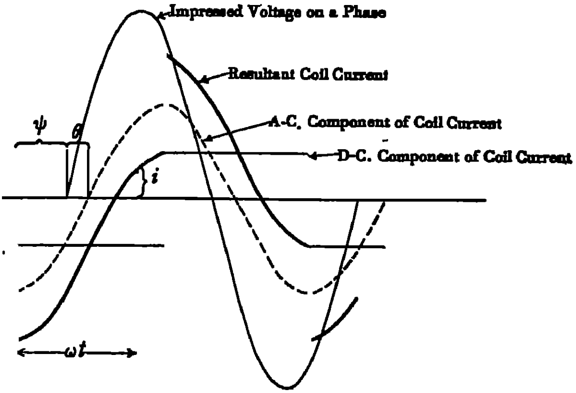

Impressed Voltage on a Phase
Resultant Coil Current
A-C. Component of Coil Current
D-C. Component of Coil Current
ψ
θ
i
ωt

FIG. 102.—Components and Resultant Current in an Armature Coil of a Synchronous Converter

Figure 102 represents the two components and the resultant current waves for coil *Q* (see Fig. 99). The converter is assumed to operate at a lagging power factor and is driven in a counter-clockwise direction. In plotting the wave shapes of currents in the coils a suitable reference curve must be used; in this particular case the phase impressed voltage is used as the reference curve.

The equation for the resultant current at any instant referred to the time at which a coil undergoes commutation is (see Fig. 102)

$$i=\sqrt{2}I_{ac(coil)}\sin\left[\omega t-(\theta+\psi)\right]-\frac{I'_{dc}}{2} \qquad (8)$$

This equation holds between the limits $\omega t=0$ and $\omega t=\pi$. In the equation θ is considered as positive when the current lags the impressed voltage and ψ is positive when the coil under consideration is located ahead (in the direction of rotation of the armature) of the central coil of a phase.

127. Armature Heating of a Synchronous Converter.—The average square of the current obtained from equation (8) is

$$D=\frac{1}{\pi}\int_0^\pi i^2_{\,dt}=\frac{I'^2_{dc}}{4}\left[\frac{8}{(\cos\theta)^2(\xi)^2q^2\sin^2\frac{\pi}{q}}+1-\frac{16\cos(\theta+\psi)}{(\cos\theta)\,\xi\pi q\sin\frac{\pi}{q}}\right] \qquad (9)$$

This equation shows that for a lagging power factor (θ positive) the r.m.s. value of the resultant current is a maximum in the coils located near the leading tapping point (ψ positive), and for a leading power factor (θ negative) the r.m.s. current is a maximum near the trailing tapping point (ψ negative). In general for a given power factor, the current in a coil has the minimum r.m.s. value when ψ is such that $(\psi+\theta)=0$ and the maximum r.m.s. value when $(\psi+\theta)$ has as large a positive or negative value as is possible in the given machine.

The average heating in a coil is proportional to the square of the r.m.s. current; it is proportional to D in equation (9). Therefore, the average heating for all the coils in a phase belt is proportional to

$$\frac{q}{2\pi}\int_{-\frac{\pi}{q}}^{\frac{\pi}{q}}D\,dx=\frac{I'^2_{dc}}{4}\left[\frac{8}{(\cos\theta)^2\xi^2q^2\sin^2\frac{\pi}{q}}+1-\frac{16}{\pi^2\xi}\right] \qquad (10)$$

128. Relative Outputs of an Armature when Operated as a Converter and as a D-C. Generator.—Let I''_{dc} = direct-current output per brush set when the armature is operated as a d-c.

generator, and let I'_{dc} = direct-current output per brush set when the armature is operated as a converter, then for equal heating

$$\frac{I''^2_{dc}}{4}=\frac{I'^2_{dc}}{4}\left[\frac{8}{(\cos\theta)^2\xi^2q^2\sin^2\frac{\pi}{q}}+1-\frac{16}{\pi^2\xi}\right]$$

and the relative output currents are

$$\frac{I'_{dc}}{I''_{dc}}=\frac{1}{\sqrt{\frac{8}{(\cos\theta)^2\xi^2q^2\sin^2\frac{\pi}{q}}+1-\frac{16}{\pi^2\xi}}} \qquad (11)$$

If we assume the efficiency 100 per cent ($\xi=1$) and the power factor unity ($\cos\theta=1$), the following results are obtained from equations (11).

Number of phases (q)	$\frac{I'_{dc}}{I''_{dc}}$
1	0.85
3	1.33
6	1.93
12	2.18

129. Armature Reaction in a Synchronous Converter.—Since motor and generator action take place in the same armature it is natural for one to expect less armature reaction in a converter than in either a d-c. generator or a synchronous motor. The a-c. components of the currents in the armature winding produce a field which revolves about the armature conductors at synchronous speed and in a direction opposite to the rotation of the armature itself. Since the armature revolves at synchronous speed, this field for a given load and excitation remains fixed in position relative to the main field poles. The field produced by the d-c. components in the armature windings also results in a reaction field, which for a given load and excitation is fixed in position with respect to the main field. The two reaction fields are in opposition because during the major portion of a cycle the current components are in opposition. For a given d-c. output the current input on the a-c. side depends on the power factor and since the armature magnetomotive forces depend on the currents it follows that the resultant armature reaction obtained in a converter depends largely on the power factor.

130. Methods for Changing the D-C. Voltage of a Synchronous Converter.—If the a-c. voltage across the rings of a converter is maintained constant, no variation in the excitation of the converter will appreciably alter the d-c. voltage. This is evident if we keep in mind that the air-gap flux in a synchronous motor remains practically constant regardless of the field current, and that the voltage developed by a d-c. generator depends on the air-gap flux. In order to change the d-c. voltage of a converter it is necessary that the phase impressed voltage of the converter be changed or that a d-c. booster be connected in series with the d-c. load. The following methods of voltage control are used:

1. Synchronous booster.
2. Induction regulator.
3. D-c. booster.
4. Split-pole field.
5. Reactance in the a-c. leads.

Synchronous booster converter (Figs. 96 and 97). The two armatures are wound for the same number of phases and poles and are mechanically fastened together by a common shaft. The armatures are electrically in series and any change in excitation of the booster field results in a change in the phase voltage across the converter armature and therefore results in a change of the d-c. voltage. The voltage of the booster armature may be made to either raise or lower the voltage.

When the booster is used for raising the phase voltage, generator action takes place in its armature, and it is mechanically driven by the converter armature. On the other hand, if the booster is used for lowering the voltage, motor action takes place in its armature, and the converter armature is driven by the booster armature. It is apparent that a change from boost to buck may result in a reversal of the armature reaction in the converter armature. For this reason the commutating field winding must be so designed as to take care of the reversal in reaction. This is accomplished by having a two-circuit winding on the commutating poles. One of the windings is connected in series with the d-c. brushes and the other winding is connected through a reversing switch across the d-c. brushes of the converter. The reversing switch is operated by the same mechanism which operates the reversing switch of the booster field.

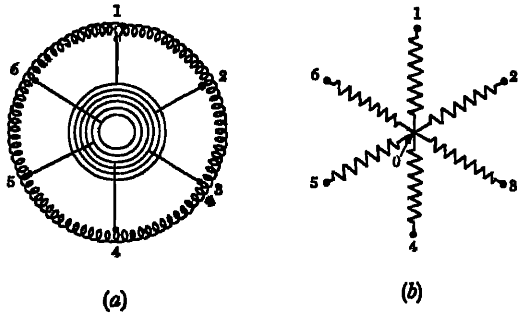
Induction regulator control.—A natural and efficient way of regulating the voltage of a converter is to insert induction regulators in the a-c. leads of the converter. This method has the disadvantage that the regulators require considerable floor space and that the first cost is high compared with that of a synchronous booster.

D-C. booster.—Another natural method for controlling the voltage is to connect a d-c. booster in the d-c. circuit. The d-c. booster requires more space and costs more than an a-c. booster. The method also has the disadvantage that two separate commutators must be maintained.

Split-pole converter.—In this type of converter a small regulating pole is located near each main pole. Two shunt field windings are required. One of the windings is wound on the main pole and the other on the regulating pole. The excitation of the two windings may be altered independently and the field form may be altered without changing the total flux per pole (main and regulating pole combined). This method for regulating the voltage is based on the fact that the voltage across the d-c. brushes depends on the total flux cut while the fundamental in the a-c. counter voltage depends on the wave shape of the field as well as on the flux cut. This method is not used a great deal on account of commutation difficulties and on account of the fact that certain transformer connections cannot be used.

Reactance method of voltage control.—In this method of d-c. voltage control inductive reactances are connected in the a-c. leads of the converter. If with this arrangement the excitation of the converter is changed, the drop across any reactance and its phase relation to the impressed voltage are changed. A leading current through the inductive reactances results in a higher collector-ring voltage than may be obtained directly from the secondary windings of the transformers supplying the converter. With the reactances inserted, over-excitation of the field of the converter results in a higher than normal d-c. voltage, and under-excitation of the field results in a lower than normal voltage.

131. Three-phase to Six-phase Transformation.—A six-phase supply may be readily obtained from a three-phase supply by the use of three stationary transformers. Several schemes for connecting the transformers are possible and it is the purpose of this article to show the principles involved in several of the schemes used.



(a) (b)

Fig. 103.—Three Hundred and Sixty Electrical-space Degrees of the Armature Winding of a Synchronous Converter

The supply voltage requirements for a balanced six-phase receiving circuit may be studied by reference to Fig. 103*a*, which represents 360 electrical space degrees of the armature winding of a six-phase converter. If oscillograms are taken of the voltage drops from 1–2, 2–3, 3–4, 4–5, 5–6, and 6–1, six waves 60 electrical time degrees out of phase are obtained. If six equal resistances are connected to 1, 2, 3, 4, 5, and 6 in star so as to form a neutral Fig. 103*b* and oscillograms taken of the voltage drop from neutral (0) to 1, 2, 3, 4, 5, and 6 in succession, voltage waves 60 time degrees apart are again obtained. Oscillograms of the voltage drop 1–3, and 6–4 show that these voltages are in phase; similarly the voltages 3–5 and 2–6 are in phase and the voltages 5–1 and 4–2 are in phase. Fig. 104 is a vector diagram showing all possible voltages in a six-phase circuit.

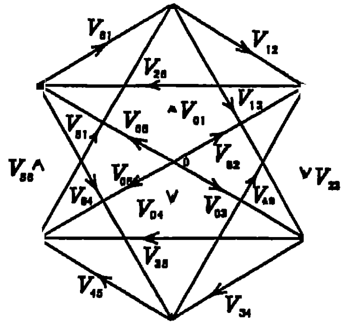


Fig. 104.—Voltage Relations in a Balanced Six-phase Load

Diametrical and double Y-connection of three transformers.—These schemes of connection are based on the fact that a single-phase circuit is no longer a single-phase circuit when the point of reference is moved from one line wire to a point located in the load. Thus in Fig. 105 when the reference point is on line 1, the voltage from 1 to 2 may be represented by a single wave. On the other hand, if the point of reference is located at *o*, two voltage waves v_{01} and v_{02}, are obtained; for a uniform impedance load these voltages are 180 degrees out of phase.

From Figs. 103 and 105 it is apparent that the voltages $(V_{10}+V_{04})$, $(V_{30}+V_{06})$, and $(V_{50}+V_{02})$ are equal and 120 time degrees out of phase. These voltages may be obtained from the secondary windings of three transformers connected as shown in Fig. 106.

The diametrical and double-Y connection differ only in that a neutral is provided in the latter scheme. The neutral may be grounded or may be used for the return circuit of the middle wire of an Edison three-wire system.

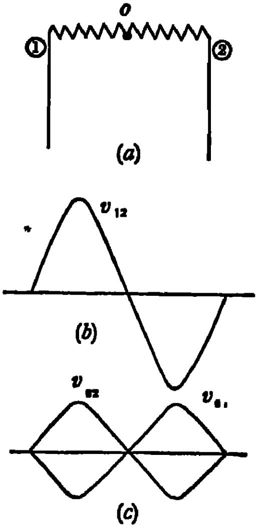


Fig. 105.—Voltage Waves of a Single-phase Load

In either the diametrical or double-Y connection the primary windings of the transformers should be connected in Y. Especially is this true when the field of the converter has a third harmonic in the space distribution of the flux. A third harmonic in the flux results in a third harmonic e.m.f. between any two tapping points on the armature located π electrical space radians apart. If the primary windings are connected in delta, a third harmonic voltage impressed across the secondary windings may result in a large circulating current through the primary windings and may produce excessive heating.

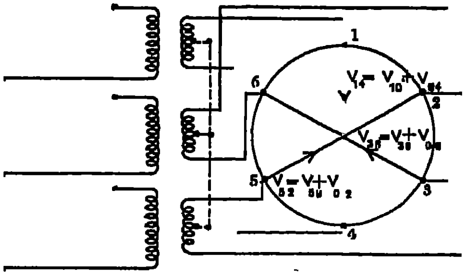


Fig. 106.—Diametrical and Double Y-Connection of Three Transformers

The double-delta connection (Fig. 107).—This scheme of connection is based on the fact that the voltages 1–3 and 6–4 are in phase and equal and may therefore be supplied by a transformer having two separate secondary windings. Similarly the voltages (3–5, 2–6) and (5–1, 4–2), respectively, may be supplied by transformers having two secondary windings. The double-delta connection is slightly more complicated than the diametrical, but the third harmonic e.m.f.'s due to a non-sinusoidal field form produce no circulating currents.

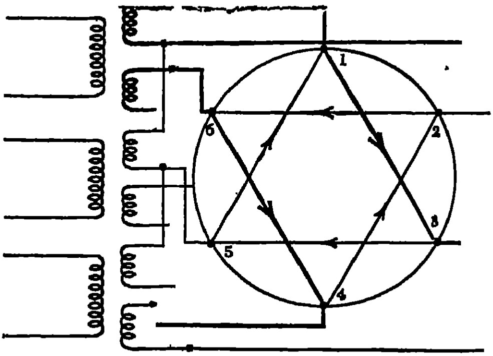


FIG. 107.—Double Delta-Connection of Three Transformers

132. Special Y-connection of Transformers for Converters Supplying an Edison Three-wire Circuit.—As previously stated, the neutral point of transformers connected in Y is sometimes connected to the middle wire of an Edison three-wire d-c. power system. The ordinary Y-connection, using one secondary winding on each transformer, cannot be used, for direct-current in the secondary winding of a transformer magnetizes the core and results in an excessive exciting current. Each transformer must have at least two secondary windings and they must be so connected that the direct-current produces no magnetization in the transformer cores. The connection shown in Fig. 108 serves the purpose. Any direct-current entering at "*O*" divides equally among the transformer windings and the d-c. magnetizing action produced by one-half of a secondary winding is balanced by the other half.

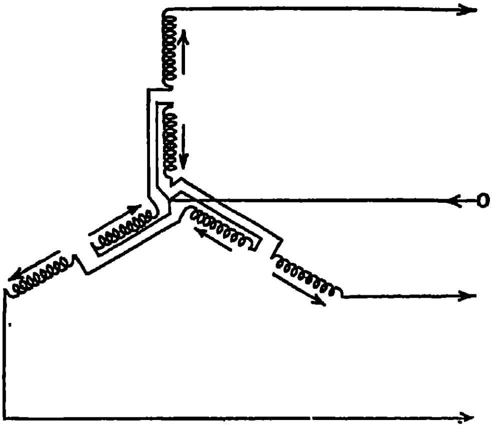


FIG. 108.—Special Y-Connection of Three Transformers

133. Methods for Starting a Synchronous Converter.—A converter is in normal operation when it runs at synchronous speed, when its d-c. polarity is correct, and when the field current is so adjusted as to obtain the desired voltage and power factor. Any electrical method of starting, in order to be practical, must bring the converter from rest to normal operation in a short time and with no excessive current requirements. The following methods of starting are used:

1. The a-c. end of the converter is operated as an induction motor and synchronized by its own synchronizing torque. (See article on starting torque of synchronous motors, Chapter IX.)
2. The d-c. end of the converter is operated as a shunt motor and the a-c. end synchronized by one of the methods used in synchronizing an alternator or a synchronous motor.
3. Auxiliary motors are coupled to the converter shaft and converter is synchronized by one of the well-known methods.

The induction motor method of starting is frequently used and is considered the best method. Polyphase power at reduced voltage (about ½ rated voltage) is supplied to the collector-rings. A revolving magnetic field is produced which induces a current in the pole-face winding and a torque results. The direction of the torque is opposite to the direction of rotation of the armature field; i.e., the armature revolves in a direction opposite to that of the armature field.

The revolving field also cuts across the shunt and series field circuits. Excessive voltages in the shunt field circuit are avoided by disconnecting the field circuit at several places. This is done by a special multiblade reversing switch. The purpose in using a reversing switch will be explained later.

Since the armature field rotates about the armature conductors at synchronous speed and the armature rotates in the opposite direction, it follows that near synchronous speed the the armature field slowly cuts across the main field poles and gives them a definite polarity for a relatively long period. This causes the armature to pull into synchronism. (See part *b*, Principle of Operation of a Synchronous motor, Chapter VIII.)

At synchronous speed, the armature field is stationary with respect to the main poles and gives them a definite polarity. Any synchronous converter or synchronous motor will operate without shunt field current, but this is never done on account of the poor power factor obtained.

When the converter has reached synchronous speed (one can tell by its sound) the field switch is closed and normal a-c. voltage is impressed across the rings. The d-c. polarity is then determined. It is entirely possible that the armature locks in synchronism in such a way as to make the polarity of a given main field pole opposite to what it should be for correct d-c. polarity. The proper polarity may be obtained by momentarily destroying the electromagnetic torque acting on the armature. This is done by reversing the field current for an instant. The main field neutralizes the armature field and the friction and windage losses cause the armature and its field to drop back and lock in with the adjacent field poles. After the armature field has slipped back one pole (one can tell by the indication of the d-c. voltmeter connected across the brushes) the shunt field switch is thrown into the running position.

Synchronous converters, when equipped with interpoles and when intended to be started by induction motor action, must be equipped with brush-lifting devices. The necessity of this additional equipment is apparent when we keep in mind that during the starting period the armature field cuts across the armature coils and produces excessive currents in those coils which are short-circuited by brushes. Brush-lifting devices lift the main brushes off the commutator and consequently prevent excessive currents in the armature coils. The current required for the field winding and for the armature voltmeter may be obtained from small brushes which remain on the commutator while the main brushes are off.

134. Values for the Losses and Efficiency of a 2000-kw. Synchronous Converter.—

RATING OF MACHINE

2000-kw.; 600-volt; 6-phase; 60-cycle; 450 r.p.m.

LOSSES AT RATED LOAD (25° C.)

Armature copper loss	=14.7 kw.
Series field loss	= 3.2 kw.
Commutating field loss	= 3 75 kw.
Shunt field loss (including rheostat)	= 5.9 kw.
Core loss	=15.6 kw.
Total friction loss	=30 0 kw.
Efficiency at rated load (unity power factor) and 25° C.	=96.5 per cent

PROBLEMS

1. Draw an armature wiring diagram of a 6-pole, six-phase synchronous converter showing the connections to the slip-rings and the location of the d-c. brushes. A ring winding, although not used in practice, may be used in illustrating the connections.

2. Why is a closed-circuit armature winding (mesh winding) essential on a synchronous converter?

3. A 4-pole, three-phase synchronous converter delivers 500 kilowatts on the d-c. side. The converter operates at a leading power factor of 97 per cent, its efficiency is 92 per cent, and the d-c. voltage is 600 volts. (*a*) Calculate the r.m.s. value of the voltage between adjacent slip-rings. (*b*) Calculate the r.m.s. value of the total current supplied to a given slip-ring.

4. A 4-pole, six-phase converter delivers 720 kilowatts at a d-c. voltage of 550 volts. The converter operates at an efficiency of 90 per cent and at a lagging power factor of 87 per cent. (*a*) Plot the waves of the a-c. and d-c. components of the current in a coil located midway between two adjacent tapping points of the armature. (*b*) Repeat (*a*) for coils located near the leading and the trailing tapping points. (*c*) Plot the resultant current waves for the coils specified in (*a*) and (*b*).

5. The following things are known about a 2000-kw., 16-pole, 600-volt (d-c.), six-phase, 60-cycle, synchronous converter:

Losses

(Measured at Rated Unity Power Factor Load—Temperature, 25° C.)

Armature copper loss	14.8 kw.
Series field loss	3.4 kw.
Commutating field loss	3.65 kw.
Shunt field loss (including rheostat)	6.9 kw.
Core loss	15.0 kw.
Total friction loss	31.6 kw.

The field of the converter is so adjusted that unity power factor is obtained on the a-c. side. Calculate the efficiency for, (*a*) 125 per cent rated load, (*b*) rated load, and (*c*) 75 per cent rated load.

6. The converter specified in Problem 5 is supplied by three transformers. Calculate: (*a*) the voltage and current ratings of the secondary windings of the transformers when the diametrical connection is used; and (*b*) the voltage and current ratings of the secondary windings when the double delta connection is used. (Assume that the converter is operated at unity power factor.)

7. A converter similar to that in Problem 5 is supplied from a 213-volt, 60-cycle, six-phase source. Reactors having effective resistances of 0.0002 ohm and inductive reactances of 0.100 ohm are connected in each lead of the six phases. The load and the excitation on the converter are so adjusted that the current in the six-phase leads is 1000 amperes. The power factor of the converter measured at the converter rings is 0.85 leading. Calculate: (*a*) the d-c. voltage of the converter when the reactors are in the circuit; and (*b*) the d-c. voltage of the converter when the reactors are short-circuited. [Assume that the power factor of the converter in part (*b*) is the same as in part (*a*).]

CHAPTER XI

MERCURY VAPOR RECTIFIERS; MECHANICAL RECTIFIERS; THERMIONIC RECTIFIERS

135. Mercury Vapor Rectifiers.—Mercury vapor rectifiers are used for charging storage batteries and for furnishing direct-current to series arc lighting systems. The power requirements for this service are usually small, but this does not mean that this type of rectifier cannot be constructed for large capacity. Mercury vapor rectifiers having capacities exceeding 1000 kilowatts are in successful operation. The voltage range of these rectifiers is from 100 to 5000 volts; the current capacity depends on the number and kind of electrodes used. The chief advantage of the mercury rectifier is that it has no mechanically moving parts. Its chief disadvantage is that the operation depends on the maintenance of a definite vacuum.

Construction of the mercury vapor rectifier.—One of the usual types of single-phase rectifiers is shown in Fig. 109. The anodes A and A' are made of graphite or iron. The cathode C is a pool of mercury. The electrode Q, also a pool of mercury, is required to produce the necessary ionization to start the main arc between the anodes and the cathode. The vacuum required in the tube must be high (approximately 0.05 mm.) and for this reason the seal where the electrodes enter the glass bulb must be perfect. The power is supplied to the rectifier by a single-phase transformer which is designed for relatively high leakage reactance. The transformer has several winding taps so that the voltage impressed across the rectifier may be readily changed.

Operation of the mercury-vapor rectifier.—In normal operation the mercury cathode is very hot (at one spot) and the anodes are relatively cool. On account of the high temperature of the mercury the cathode becomes a source of mercury vapor and also a source of electrons. The bombardment of the mercury molecules by the electrons results in ionized mercury vapor. A current can flow from either anode to the cathode through the ionized mercury vapor, but it cannot flow from the cathode to the anodes. This is evident if we keep in mind that very few electrons are emitted from the relatively cool anodes. The operation of the rectifier is as follows: When the connecting lead 1 is positive, the current enters electrode *A* and leaves the bulb through electrode *C* (the electrons flow in the opposite direction) and returns to the transformer through the d-c. load. During the next half cycle, lead 2 is positive, the current now enters electrode *A′* and returns through electrode *C* and the connected load. Both halves of the cycle are rectified. The purpose of the high leakage reactance of the transformer is to smooth out the unidirectional current wave and to maintain the ionization of the mercury vapor when the impressed voltage passes through the zero values in a cycle.

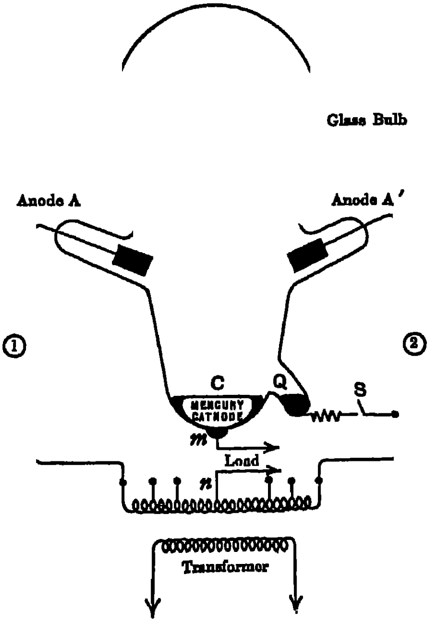


Fig. 109.—Mercury Vapor Rectifier

To put the rectifier in operation, a-c. power is applied to the primary winding of the transformer, switch *S* is closed, and the bulb is tilted so that an arc is formed between the electrodes *C* and *Q*. This arc instantly fills the bulb with ionized mercury vapor and the current is established between the main electrodes. Switch *S* is then opened and the bulb is tilted back in the normal operating position. During the starting period the impedance of the circuit *m* to *n* must be sufficient to prevent too rapid heating of the bulb.

The efficiency of a given rectifier bulb and necessary equipment is a function of the impressed voltage; it is high when the voltage is high, the reason for this being that for a given current the losses in the mercury arc are independent of the operating voltage and the power output is directly proportional to the operating voltage. The efficiencies of mercury rectifiers range between 68 and 97 per cent, depending on the operating voltage.

136. Vibrating Mechanical Rectifiers.—Mechanical rectifiers are used for charging automobile and radio batteries. The simplest form of half-wave rectifier is shown in Fig. 110. The main parts are a vibrator *G*, a permanent magnet *M*, an a-c. electromagnet *C*, a contactor *F*, and a step-down transformer *T*. The secondary circuit through the contactor *F* is closed every other half cycle and a unidirectional current is forced through the battery. The efficiency of this type of rectifier cannot be very high, for the simple reason that during the major portion of a cycle it is doing no useful work. However, its simplicity and low first cost partly compensate for the low efficiency.

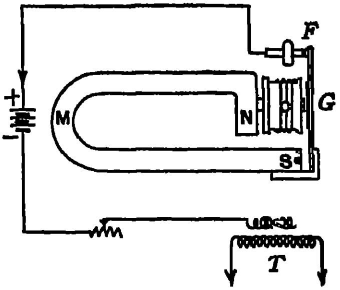


FIG. 110.—Mechanical Rectifier

137. Thermionic Rectifiers (Tungar and Kenotron).—Both the tungar and the kenotron rectifiers consist of an evacuated glass bulb in which two electrodes are located (see Fig. 111). One of the electrodes, the filament *F*, is operated at a high temperature and emits electrons. The other electrode, the plate *P*, is relatively cool. During the positive half of the cycle the electrons emitted by the filament are attracted by the plate and a flow of current is established between them. During the negative half of the cycle the plate is charged negatively, the electrons emitted by the filament are repelled, and no current flows.

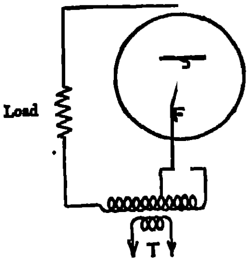


FIG. 111.—Thermionic Rectifier

The tungar rectifier is operated at low voltage and relatively high current (15–6 volts and 3–6 amperes) while the kenotron must be operated at high voltages and small current (100,000 volts and 100 milliamperes). A very high vacuum is used in the kenotron and the current conduction through the tube depends on the electrons emitted by the filament. The vacuum in a tungar rectifier is not so high; the tube contains argon gas which becomes ionized and serves as the principal carrier of the current through the tube.

CHAPTER XII

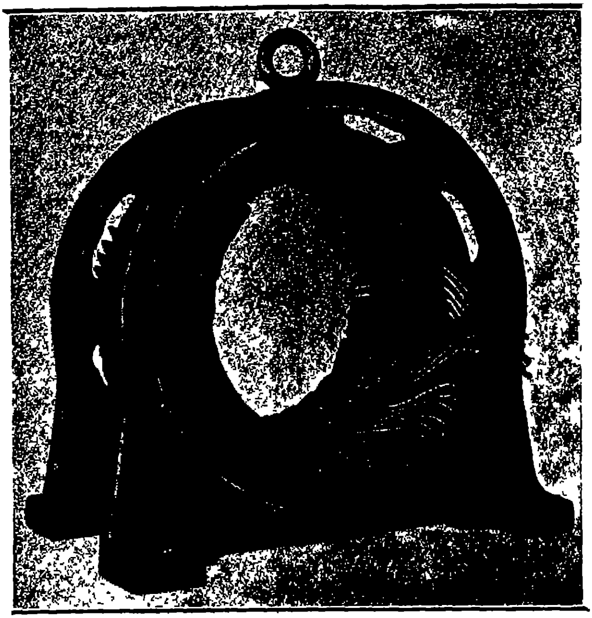
POLYPHASE INDUCTION MOTOR

An induction motor is an a-c. motor in which at least a portion of the current in the rotor is due to induction. Its speed characteristic resembles that of an ordinary d-c. shunt motor. Its efficiency and power factor depend on the design, the load, and the method of speed control used.

A properly designed induction motor is very rugged and may be adapted to almost every industrial requirement. The mere fact that these motors require no sliding contacts makes it possible to operate them under conditions where other motors are unsatisfactory.

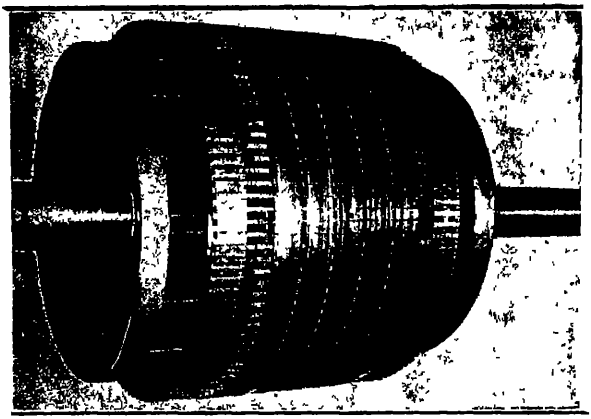
138. Windings of a Polyphase Induction Motor.—A polyphase induction motor requires two polyphase windings. One of the windings is located on the stator (see Fig. 112) and is similar in every way to the armature winding of a polyphase generator or synchronous motor. The other winding is located on the rotor and has the form of a squirrel cage (Fig. 113) or the form of an ordinary polyphase winding connected to collector-rings (Fig. 114). The a-c. power is usually supplied to the motor through the stator winding and for this reason the stator winding is often called the *primary winding*. The electrical power in the rotor winding is due to induction. In fact the rotor winding acts like the secondary winding of a transformer and for this reason it is often called the *secondary winding*. Both windings are wound for the same number of poles, but the number of phases on the windings may be different. For example, a three-phase squirrel-cage induction motor has three phases on the stator while the number of phases on the rotor is equal to the number of rotor slots located in 360 electrical space degrees of the stator winding.

139. Principle of Operation of a Polyphase Induction Motor.—Fig. 115 represents the simplest kind of two-phase induction motor. The stator winding is represented by two phases, A and B, located



(By the courtesy of the Westinghouse Co.)

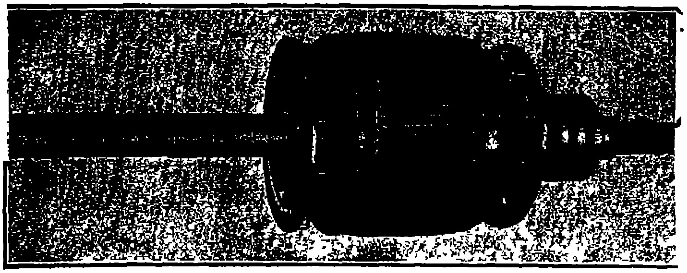
Fig. 112.—Partly Wound Stator of an Induction Motor



(By the courtesy of the Westinghouse Co.)

Fig. 113.—Squirrel-cage Rotor for an Inductive Motor

90 electrical space degrees apart. A current entering terminal 1 of phase A produces a field to the right, while a current entering terminal 1 of phase B produces a field down.



(By the courtesy of the Westinghouse Co.)

FIG. 114.—Wound Rotor for an Induction Motor

If the current entering terminal 1 of phase A is of the form

$$i_a = \sqrt{2}\, I \cos(\omega t)$$

and that entering terminal 1 of phase B of the form

$$i_b = \sqrt{2}\, I \cos(\omega t - 90)$$

it follows that at the time $t=0$ the magnetic field is produced by winding A alone and the direction of the field is to the right. Similarly at the time when $\omega t = 90$ the field is produced by phase B alone and is directed downward. When $\omega t = 45$ the fields produced by phases A and B are equal and each field has 0.707 of its maximum strength making the resultant strength at this instant equal to $\sqrt{0.707^2 + 0.707^2} = 1$; that is, the resultant field is equal to that produced by one phase alone when the current in the phase is passing through a maximum value. The resultant fields for other values of ωt are readily determined. By actually carrying out this process it is found that the resultant field is constant in magnitude and that it revolves 360 elec-

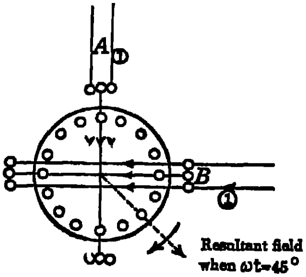


FIG. 115.—Two-phase Induction Motor

trical space degrees during the time of a complete cycle of the current in a primary phase.

The revolving field produced by the stator currents cuts the rotor inductors and sets up currents in them. The torque developed depends on the magnetic reactions between the stator and the rotor circuits. More explicitly, the torque of an induction motor depends on the air-gap flux, the rotor current, the rotor power factor, and the radius of the rotor.

CHAPTER XIII

ANALYSIS OF THE PERFORMANCE OF A POLYPHASE INDUCTION MOTOR

140. Rotor: Slip: E.M.F.; Frequency; Reactance; and Resistance.—

Rotor slip.—In an ordinary induction motor torque is developed only when the speed of the rotor is lower than the speed of the revolving magnetic field produced by the currents. The speed of the revolving magnetic field is strictly proportional to the frequency of the current in the stator winding and may be calculated by the well-known relation $n_1=\frac{2f_1}{p_1}$. The rotor speed n_2 depends on n_1, the mechanical load on the rotor, the constants of the motor, and on the voltage impressed on the stator. The ratio

$$s=\frac{n_1-n_2}{n_1} \tag{1}$$

is called the slip of the rotor.

Rotor e.m.f.—The revolving air-gap field induces voltages in both the stator and the rotor phases. If the speed of the rotor is zero the relative speed between the rotor inductors and the revolving field is n_1 (synchronous speed) revolutions per second and the rotor phase e.m.f. has a value of E_2 volts. If the speed of the rotor is equal to the speed of the magnetic field and their directions of rotation are the same the relative speed between the rotor inductors and the magnetic field is zero and therefore the e.m.f. in the rotor is also zero. Since the air-gap flux is nearly constant in magnitude (see Article 144) it follows that the e.m.f. generated in the rotor is directly proportional to the difference between the speed of the revolving field and the speed of the rotor and that this e.m.f. may be expressed by the relation

$$E_2'=k(n_1-n_2)$$

If n_2 is zero the e.m.f. becomes

$$E_2=kn_1$$

and therefore for any rotor speed n_2 the e.m.f. may be calculated by the relation

$$E'_2=\left(\frac{n_1-n_2}{n_1}\right)E_2=sE_2 \qquad (2)$$

Rotor frequency.—The frequency of the voltage induced in the rotor of an induction motor also depends on the difference between the speed of the air-gap field and the speed of the rotor. The two limiting values of speed are obtained when the rotor is locked and when the rotor is driven in the direction of rotation of the air-gap field at synchronous speed. When the rotor is locked the air-gap field cuts the rotor inductors at synchronous speed and the frequency of the voltage induced in its windings is equal to the frequency of the voltage impressed on the stator windings. At synchronous speed the rotor frequency is zero. For any rotor speed n_2 the rotor frequency may be calculated by the relation

$$f_2=\frac{p_1}{2}(n_1-n_2)$$

Since

$$f_1=\frac{p_1}{2}n_1$$

it follows that

$$f_2=\left(\frac{n_1-n_2}{n_1}\right)f_1=sf_1 \qquad (3)$$

Rotor phase leakage reactance.—The reactance of a circuit depends on the frequency, and for this reason the leakage reactance of a rotor phase of an induction motor is not constant, but depends on the speed of the rotor. If the rotor is locked the rotor leakage reactance has its maximum value and may be calculated by the relation

$$x_2=2\pi f_1L_2$$

where $f_1=f_2=$primary impressed frequency and $L_2=$rotor phase leakage reactance. For a rotor speed n_2 the leakage reactance is

$$x'_2=2\pi(sf_1)L_2$$

and therefore

$$x'_2=sx_2 \qquad (4)$$

Rotor phase resistance.—The effective resistance of a rotor phase depends on the nature of the winding and on the frequency of the rotor current. If the rotor is operating near synchronous speed the rotor phase effective resistance is equal to the d-c. resistance. If the rotor is locked, the rotor phase effective resistance is approximately 25 per cent greater than the d-c. resistance. Ordinarily an induction motor operates at a small slip and the rotor effective resistance may be assumed equal to the d-c. resistance.

141. Speed of the Rotor Reaction Field Relative to the Stator Field.—The magnetic reaction produced by the rotor currents rotates about the rotor inductors at a speed which is determined by the slip of the rotor and the number of poles for which the rotor is wound. To obtain the best torque conditions the rotor and the stator must be wound for the same number of poles. Assuming that this is so the speed of the rotor reaction field with reference to the rotor inductors may be calculated by the relation

$$n'_2 = (sf_1)\frac{p_1}{2} = sn_1$$

The direction of rotation of the reaction field depends on the phase rotation of the rotor currents. Since the stator field continuously slips forward with respect to the rotor inductors it follows that the phase rotation of the currents in the rotor phases is in the direction of rotation of the rotor and that the rotor reaction field rotates in the same direction as the rotor. The speed of the rotor inductors relative to the stator inductors is

$$n_2 = (1-s)n_1$$

and therefore the resultant speed of the rotor reaction field relative to the stator inductors is

$$(1-s)n_1 + sn_1 = n_1 \tag{5}$$

That is, the reaction field due to the rotor currents alone and the field due to the stator currents alone rotate at the same speed and in the same direction. The resultant magnetizing action of the stator and rotor windings combined produces the air-gap flux.

142. Comparison of the Circuits of an Induction Motor with Those of a Stationary Transformer.—The magnetic reaction between the stator and the rotor of an induction motor is identical to the reaction existing between the primary and the secondary windings in a stationary transformer. That this is so follows from the fact that the air-gap flux ϕ_M (the mutual flux) is a function of the vector difference of the stator ampere-turns N_1I_1 and the rotor ampere-turns N_2I_2 (the two magnetomotive forces rotate at the same speed and in the same direction). In addition to this similarity it might be pointed out that a sine-wave (in the space distribution) revolving magnetic field of ϕ_M lines cutting across a full-pitch armature coil at a speed of $2\pi\left(\frac{p_1}{2}\right)n_1$ electrical space radians per second, induces the same e.m.f. as a sine-wave alternating field having a maximum value of ϕ_M lines linking the given armature coil at a speed of $2\pi f_1$ electrical time radians per second. For these reasons the mathematical analysis of the performance of an induction motor is similar to the analysis of a stationary transformer.

143. Rotor Current; Rotor Power Factor; and Rotor Power.—For a slip s, the current in a rotor phase of an induction motor is

$$I_2=\frac{sE_2}{\sqrt{r_2^2+(sx_2)^2}}\quad\text{(machine running)}\qquad(6)$$

and the power factor of the rotor circuit is

$$\cos\theta_2=\frac{r_2}{\sqrt{r_2^2+(sx_2)^2}}\qquad(7)$$

where sE_2 is the e.m.f. generated in a rotor phase, r_2 is the resistance of a rotor phase, and sx_2 is the leakage reactance of a rotor phase.

If the numerator and the denominator of equation (6) are divided by s, the current I_2 is unchanged. but the equation now has the form

$$I_2=\frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2+x_2^2}}\quad\text{(machine locked)}\qquad(8)$$

and the rotor power factor

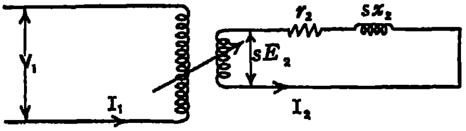
$$\cos\theta_2=\frac{\frac{r_2}{s}}{\sqrt{\left(\frac{r_2}{s}\right)^2+x_2^2}}=\frac{r_2}{\sqrt{r_2^2+(sx_2)^2}}\qquad(9)$$

is also unchanged.

The resistance r_2 in the expressions (6–9) is the effective phase resistance of the rotor when the slip is s.

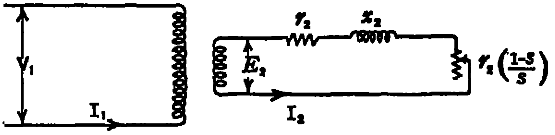
These equations show that an induction motor operating at a given slip and with the rotor short-circuited has the same rotor current, the same rotor power factor, and hence the same rotor winding magnetic reaction as an identical motor with the rotor locked (see Fig. 116) and an external resistance inserted in each phase of value $\frac{r_2}{s}-r_2=\left(\frac{1-s}{s}\right)r_2$. The change in rotor resistance due to change in frequency is here neglected.

Since the secondary magnetic reactions are equal and bear the same time phase relations (same secondary power factors) relative to the stator impressed voltages, it follows (neglecting change in rotor iron loss) that both machines in Fig. 116 have equal primary power factors and equal primary currents. The electrical inputs to the stator windings are approximately equal and the power transferred across the air-gap is the same in each machine.



Short-circuited Revolving Rotor

(a)



Locked Rotor

(b)

Fig. 116.—Circuits of an Induction Motor

If F_1 is the tangential force on the rotor and D_1 is the peripheral velocity of the revolving field around the rotor, then the total power transferred across the air-gap of each machine is F_1D_1. In the revolving machine the actual mechanical power developed is F_1D_2, where D_2 is the peripheral velocity of the rotor at the operating speed. In the locked machine no mechanical power is developed by the rotor and therefore the total power transferred across the air-gap shows up in the form of electrical power in the rotor winding and as electrical losses in the rotor iron.

The electrical power developed in the rotor of the revolving machine is equal to the copper loss in its windings and is equal to

$$qr_2I_2^2=qsE_2I_2\cos\theta_2 \text{ watts} \qquad (10)$$

The total copper loss in the rotor of this machine may also be expressed by the relation

$$qr_2I_2^2=\frac{(F_1D_1-F_1D_2)746}{550}-\text{rotor iron loss} \quad . \qquad (11)$$

where q=number of rotor phases, F_1 is expressed in pounds, and D_1 and D_2 are expressed in feet per second.

The electrical power developed in the rotor winding of the locked machine is

$$q\left(\frac{r_2}{s}\right)I_2^2=qE_2I_2\cos\theta_2 \text{ watts} \qquad (12)$$

From equations (10) and (12) and the fact that the power actually transferred across the air-gap is the same for each machine it follows that the mechanical power developed in the rotor of the running machine is

$$qE_2I_2\cos\theta_2-qr_2I_2^2=q(1-s)E_2I_2\cos\theta_2 \quad \text{(approximately)} \qquad (13)$$

or

$$q(1-s)E_2I_2\cos\theta_2=q\left(\frac{1-s}{s}\right)r_2I_2^2 \quad \text{(approximately)} \qquad (14)$$

144. Exactly Equivalent Circuit of an Induction Motor.— The current input I_1 to a primary phase of an induction motor may be resolved into two components—namely, the no-load current I_N required to produce the mutual flux and the load current I_{21} required to balance the magnetizing action of the secondary winding. The no-load current on account of the presence of the air-gap in the magnetic circuit is approximately 25–40 per cent of the rated full-load current compared with approximately 7 per cent for a stationary transformer. The load component of the primary current is determined in the same way as in a transformer, i.e., $I_{21}=\frac{N_2}{N_1}I_2$. The total primary current is

$$\underset{\cdot}{I}_1=\underset{\cdot}{I}_N+\underset{\cdot}{I}_{21} \qquad (15)$$

The voltage impressed on a primary phase depends on the voltage due to the air-gap flux, the phase resistance, the phase leakage reactance, and the primary current. The voltage across a primary phase may be calculated by the relation

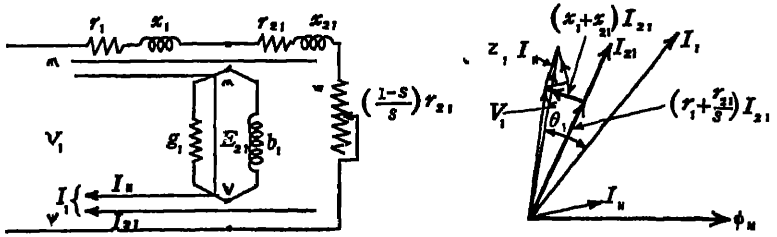
$$V_1 = E_1 + (r_1 + jx_1)I_1 \tag{16}$$

where E_1 is the voltage due to the air-gap flux and $(r_1+jx_1)I_1$ is the voltage drop due to the leakage impedance of the primary winding. Since $(r_1+jx_1)I_1$ is small (not so small as in a transformer) it follows that

$$V_1 = E_1 \quad \text{(approximately)}$$

In other words, for a given primary voltage and frequency the air-gap flux is nearly constant.

From these relations it is apparent that the circuits and the voltage and current relations in an induction motor correspond in every way to those in a transformer. A given phase of an induction motor may therefore be represented by the diagram shown in Fig. 117*a*.



(*a*) (*b*)

FIG. 117.—Exactly Equivalent Circuit of an Induction Motor and Vector Diagram

145. Approximate Equation for the Torque Developed by an Induction Motor.—The total power entering the rotor of an induction motor may be expressed by the relation

$$qE_2I_2 \cos\theta_2 = qE_{21}I_{21} \cos\theta_{21} \text{ watts}$$

and the total torque developed by this power may be expressed by the relation

$$T = \frac{550}{746}\, q\, \frac{(E_{21}I_{21} \cos\theta_{21})}{2\pi n_1} \tag{17}$$

where T = torque in pound-feet;
n_1 = synchronous speed;
and q = number of rotor phases (assumed equal to the number of stator phases).

Equation (17) may be modified so as to obtain the torque in terms of the primary phase voltage and the constants of the motor. Thus, from Fig. 117*a* it is apparent that

$$E_{21}=V_1-z_1I_{21}\quad\text{(approximately)}\qquad(18)$$

(The drop due to the no-load current is neglected).

Since

$$I_{21}=\frac{E_{21}}{z_{21}}\quad\text{(exactly)}\qquad(19)$$

it follows from equation (18) that

$$E_{21}=V_1\left(\frac{z_{21}}{z_{21}+z_1}\right)$$

and therefore

$$E_{21}=\frac{V_1\sqrt{\left(\frac{r_{21}}{s}\right)^2+(x_{21})^2}}{\sqrt{\left(\frac{r_{21}}{s}+r_1\right)^2+(x_{21}+x_1)^2}}\quad\text{(approximately)}\qquad(20)$$

The power factor

$$\cos\theta_{21}=\frac{\frac{r_{21}}{s}}{\sqrt{\left(\frac{r_{21}}{s}\right)^2+(x_{21})^2}}\quad\text{(exactly)}\qquad(21)$$

Substituting (19), (20), and (21) in (17), we obtain

$$T=\frac{550}{746}\,\frac{qV_1^2}{2\pi n_1}\left[\frac{s\,r_{21}}{(r_1s+r_{21})^2+s^2(x_1+x_{21})^2}\right]\quad\text{(approximately)}\qquad(22)$$

Equation (22) shows that for a motor operating at a given slip and frequency the torque developed is approximately proportional to the square of the impressed voltage.

146. Slip at which the Maximum Torque Occurs.—For a given impressed voltage and for a given set of motor constants the slip at which the maximum torque occurs may be obtained by differentiating equation (22) with respect to the slip (r_1, r_{21}, x_1, x_{21}, n_1, and V_1 are assumed constant) and equating the expression obtained to zero. Thus

$$\frac{d}{ds}\left(\frac{sr_{21}}{(r_1 s+r_{21})^2+s^2(x_1+x_{21})^2}\right)=0$$

or

$$s=\frac{r_{21}}{\sqrt{r_1^2+(x_1+x_{21})^2}} \qquad (23)$$

This value of slip substituted in equation (22) gives for the maximum running torque

$$T_{\text{max.}}=\frac{550}{746\ 2\pi n_1}\ \frac{qV_1^2}{2\left\{r_1+\sqrt{r_1^2+(x_1+x_{21})^2}\right\}} \qquad (24)$$

Equation (23) shows that the slip at which maximum torque occurs depends on the rotor resistance.

Equation (24) shows that the maximum running torque is independent of the rotor resistance, provided the value of the rotor resistance r_2 referred to the primary winding is such as to make the value of the slip calculated by equation (23) lie between the limits of 0 and 1.

147. Rotor Phase Resistance Required to Obtain the Maximum Torque at Starting.—The value of the rotor phase resistance required to obtain the maximum torque at starting may be obtained by substituting unity for the slip in equation (23). Thus the resistance

$$r_{21}=\sqrt{r_1^2+(x_1+x_{21})^2} \qquad (25)$$

gives the maximum torque at starting.

148. Torque Slip Curves for an Induction Motor (Fig. 118).—Curves A and B for a given motor may be determined by plotting equation (22). Curve A is obtained by substituting for r_{21} the actual rotor phase resistance referred to the stator winding and curve B is obtained by substituting for r_{21} the value $\sqrt{r_1^2+(x_1+x_{21})^2}$. The maximum values of the torques are equal, but the slips at which the maximum torques occur are different.

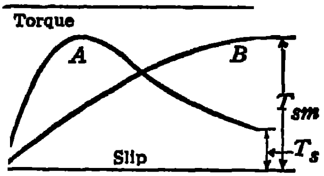


FIG. 118.—Torque Slip Curves for an Induction Motor

149. Predetermination of the Performance Curves of an Induction Motor.—The following quantities may be calculated or determined by test (see tests in Chapter XIV):

r_1 = primary effective phase resistance;
r_{21} = secondary effective resistance referred to the primary winding;
x_1 = primary phase leakage reactance;
x_{21} = secondary leakage reactance referred to the primary winding for a slip $s=1$;
I_N = no-load current;
$\cos\theta_N$ = no-load power factor;
$W_{f.w.}$ = friction and windage losses in the rotor for different values of slip;

and

$W_{i.r.}$ = iron loss in the rotor for different values of slip.

For a given impressed voltage V_1 and for a given slip s, the rotor input may be calculated by the relation

$$P_{ri}=q\left(\frac{r_{21}}{s}\right)I_{21}{}^2+W_{ir} \text{ watts} \tag{26}$$

This relation requires that I_{21} be known. Since I_N lags V_1 by nearly 90 degrees, I_{21} may be calculated to a close degree of approximation by the relation

$$I_{21}=\frac{V_1-z_1I_N(\text{numerical difference})}{\sqrt{\left(r_1+\frac{r_{21}}{s}\right)^2+(x_1+x_{21})^2}} \tag{27}$$

This equation follows from the vector diagram Fig. 117*b* in which the voltage drop due to the currents I_{21} and I_N are represented.

The mechanical output of the rotor may be calculated by the relation

$$P_{ro}=qr_{21}\left(\frac{1-s}{s}\right)I_{21}{}^2-P_{f.w.} \text{ watts} \tag{28}$$

The rotor power factor is

$$\cos\theta_2=\frac{r_{21}}{\sqrt{(r_{21})^2+(sx_{21})^2}}=\cos\theta_{21} \tag{29}$$

The primary power factor may be determined from the vector diagram, and the primary current may be calculated by the relation

$$\underset{\cdot}{I}_1=\underset{\cdot}{I}_{21}+\underset{\cdot}{I}_N \tag{30}$$

The total input to the primary winding is

$$qV_1I_1 \cos\theta = \text{Output} + \text{losses} \tag{31}$$

and the efficiency

$$\xi = \frac{\text{Output}}{\text{Output} + \text{losses}} 100 \tag{32}$$

The torque developed for different values of slip may be calculated by the relation

$$T = \frac{550}{746} \frac{qV_1{}^2}{2\pi n_1} \left[\frac{sr_{21}}{(r_1s + r_{21})^2 + s^2(x_1 + x_{21})^2} \right] \tag{33}$$

PROBLEMS

1. The stator of a certain 25-cycle induction motor is wound for 12 poles. Calculate: (*a*) the speed of the revolving magnetic field in r.p.m.; (*b*) the number of magnet poles formed on the rotor; (*c*) the frequency of the currents induced in the rotor at standstill; (*d*) the frequency of the currents in the rotor when the slip is 8 per cent; and (*e*) the speed of the rotor when the slip is 8 per cent.

2. (*a*) Why is the no-load current of an induction motor high compared with the no-load current of a power transformer?

(*b*) Is the presence of a high no-load current objectionable to, (1) the consumer, and (2) the central station?

3. Show that the magnetic reaction produced by the rotor currents of an induction motor is identical with the magnetic reaction produced by the secondary current in an ordinary power transformer.

4. Show that the total electrical power developed in the rotor winding of an ordinary induction motor with no external resistances connected in the circuit, is equal to the total copper loss in the rotor winding.

5. Derive an equation for the total power input to the rotor of an ordinary polyphase induction motor.

6. Two identical polyphase induction motors are connected to the same power supply. One of the motors is operated at a slip of 5 per cent and with its rotor winding short-circuited. The rotor of the other motor is locked and resistances of $19r_2$ ohms added externally to each rotor phase (r_2 is the rotor phase resistance). Determine the relative values of: (*a*) the rotor e.m.f.'s; (*b*) the rotor currents; (*c*) the power factors of the rotor circuits; (*d*) the stator currents; (*e*) the power transferred across the air-gaps to the rotors; (*f*) the power losses in the rotor circuits; and (*g*) the power loss in the resistances ($19r_2$) and the total mechanical output of the revolving machine.

7. The following measurements were made on a 3-horse power, 4-pole, three-phase, 110-volt, 60-cycle induction motor;

D-c. resistance between stator terminals at 40° C. = 0.360 ohm.

D-c. resistance between rotor terminals (the machine has a three-phase rotor) at 40° C. = 0.58 ohm

VOLTAGE RATIO TEST

(Rotor Stationary and Open-circuited)

Stator voltage between terminals	99.7
Rotor voltage between terminals	110.0

NO-LOAD TEST

(Rotor Running Free and Short-circuited, Temperature, 40° C.)

Terminal voltage	110 volts
No-load current	6.0 amperes
Frequency	60 cycles

Total power input, 277 watts (173 watts core loss and 104 watts friction + windage).

IMPEDANCE TEST

(Rotor Short-circuited and Locked, Temperature, 40° C.)

Terminal voltage	18 volts
Frequency	60 cycles
Current per terminal	13 amperes
Total power input (three phases)	250 watts

(*a*) Calculate the Y-phase constants of the motor referred to a reference temperature of 75° C. (Assume that the secondary (rotor) leakage reactance referred to the primary (stator) is equal to the primary leakage reactance.)

(*b*) Calculate the delta-phase constants of the motor referred to 75° C.

(*c*) Predetermine the torque slip curve. (Assume that the rotor is short-circuited and that the temperature of the machine is 75° C.)

(*d*) Calculate the maximum torque and the corresponding slip of the motor (rotor short-circuited).

(*e*) Calculate the starting torque (rotor short-circuited).

(*f*) Calculate the value of the resistance which must be connected externally in each rotor phase to obtain the maximum torque at starting.

(*g*) Predetermine the torque slip curve for the condition when the rotor resistance is such as to obtain the maximum torque at starting.

(*h*) Plot curves showing the relation between efficiency and horse power output for the conditions in parts (*c*) and (*g*).

8. (*a*) The motor in Problem 8 is operated at rated voltage, frequency and load and with its rotor short-circuited. Calculate the power factor.

(*b*) Calculate the power factor of this motor when it is operated at rated load and frequency but at 125 per cent rated voltage. (Assume that a 25 per cent increase in the voltage increases the no-load current 42 per cent and the no-load loss 30 per cent.)

CHAPTER XIV

CIRCLE DIAGRAM FOR A POLYPHASE INDUCTION MOTOR

150. The Approximately Equivalent Circuit of an Induction Motor.—The approximate circuit (Fig. 119) of an induction motor may be obtained from the exactly equivalent circuit (Fig. 118) by combining the primary and secondary leakage impedances and by connecting the exciting circuit directly across the supply. From the circuit obtained it follows that the voltage impressed on a primary phase may be expressed by the relation

$$\dot{V}_1 = (R_1 + jX_1)\dot{I}_{21} + r_{21}\left(\frac{1-s}{s}\right)\dot{I}_{21} \quad \text{(approximately)} \qquad (1)$$

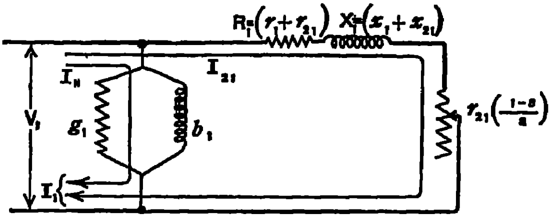


FIG. 119.—Approximately Equivalent Circuit of an Induction Motor

where $(R_1+jX_1)\dot{I}_{21}$ is the equivalent leakage impedance drop in a phase of the motor and the drop $\left[r_{21}\left(\frac{1-s}{s}\right)I_{21}\right]$ when multiplied by the current I_{21} is a function of the power output of the motor.

If the voltage V_1 across a phase of the motor is constant and if the equivalent resistance R_1 and the equivalent leakage reactance X_1 of a phase of the motor are also constant, it is possible to represent the voltage drops and the currents by circle diagrams.

Consider the voltage drop-current vector diagram (Fig. 120) which represents voltage drops and current I_{21} for the circuit in Fig. 119. The angle $ABC = 180 - \beta_1$ and is constant provided the ratio $\frac{X_1}{R_1}=\tan\beta_1$ is constant. If the vector V_1 is constant in magnitude and is fixed in position, point B will always fall on the arc of the circle $ABCD$ (equal inscribed angles subtended by the same chord) regardless of the value of the slip. Thus, an increase in the slip reduces the value of the resistance $r_{21}\left(\frac{1-s}{s}\right)$ the current I_{21} increases and point B moves closer to point C. If the slip decreases, the resistance $r_{21}\left(\frac{1-s}{s}\right)$ increases and point B will be located nearer point A. This diagram may be called the voltage drop circle diagram.

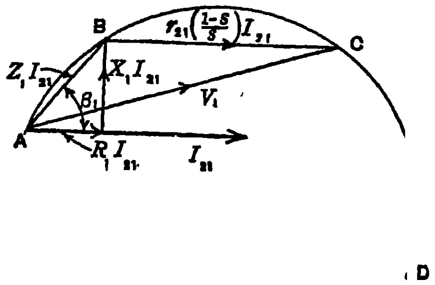


FIG. 120.—Circle Diagram for a Circuit Having Constant Reactance and Constant Impressed Voltage

A current circle diagram may be obtained by dividing the voltage drops AB, BC, and AC by the impedance $Z_1=\sqrt{R_1^2+X_1^2}$ of a phase. A voltage drop vector divided by an impedance results in a current vector which lags the given voltage vector by an angle β_1 degrees. By actually carrying out the division the diagram shown in Fig. 121 is obtained.

In this diagram the triangle AEF is similar to the triangle ABC. The angle AEF is therefore equal to the angle ABC and is constant as long as the impedance angle β_1 is constant. The point E is therefore always located on the arc of the circle $AEFG$ regardless of value of the slip s.

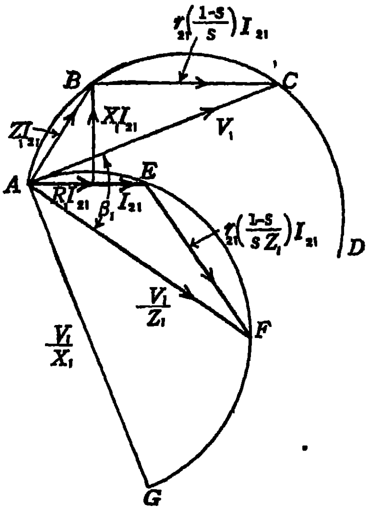


FIG. 121.—Voltage and Current Circle Diagram for a Circuit Similar to that of an Induction Motor

The vector AE represents the secondary phase current referred to the primary winding when the machine is running and the vector AF represents the secondary current referred to the primary winding when the rotor is locked; i.e., when the expression $r_{21}\left(\frac{1-s}{s}\right)=0$. The vector $AG=\frac{V_1}{X_1}$ is the diameter of the current circle and represents the current in the secondary winding referred to the primary when the total equivalent resistance of the motor is zero. Since the voltage drop circle diagram is based on the assumption that the supply voltage, the winding resistances, and the winding leakage reactances are constant, it follows that the current circle diagram which is obtained from the voltage circle diagram is based on the same assumptions. The current circle is useful in predetermining the performance curves of an induction motor.

151. Complete Circle Diagram for an Induction Motor.—The current input to a primary phase of an induction motor may be expressed by the relation

$$\dot{I}_1=\dot{I}_{21}+\dot{I}_N \tag{2}$$

where I_{21} represents the component of the primary current which is due to the load on the motor and I_N represents the component required to produce the mutual flux (air-gap flux) and supply the no-load losses in the motor. For a given impressed voltage the component I_N is practically constant and the component I_{21} may be obtained from a current circle diagram. Fig. 122 represents

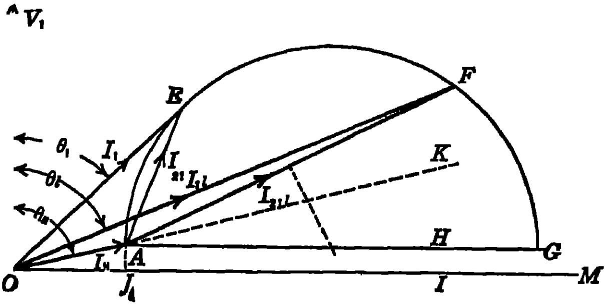


FIG. 122.—Current Circle Diagram of an Induction Motor

the complete voltage-current diagram taking into account both components of the current.

152. Tests Necessary for Determining the Circle Diagram.— No-load test.—In this test the motor is operated at no-load and with its rotor short-circuited. Measurements of the phase power and phase current are made at rated voltage and frequency. These measurements enable one to calculate the phase angle θ_N required to properly locate I_N on the diagram.

Leakage impedance test.—This test requires that the rotor be short-circuited and locked so it cannot revolve. Measurements are made of the phase voltage, phase current, and phase power at rated frequency. The voltage impressed on the motor depends on its impedance. Small motors may be tested at rated voltage while large ones, on account of the large current required, must be tested at a lower voltage.

From the observed values of current, voltage, and power the current I_{1l} (the stator current at rated voltage and rated frequency with the rotor short-circuited and locked) and the angle θ_{1l} may be calculated. Thus the current $\underset{\cdot}{I}_{2l}+\underset{\cdot}{I}_N$ is almost directly proportional to the impressed voltage and therefore

$$I_{1l} \text{ (at rated voltage)} = I_{1t}\left(\frac{V_1}{V_{1t}}\right) \text{ (approximately)}$$

where V_{1t} and I_{1t} represent the voltage and the current obtained in the test and V_1 represents the rated phase voltage of the motor.

Since the copper losses depend on the square of the currents and therefore on the square of the impressed voltage it follows that the component FH (the component in phase with the impressed voltage which is due to the copper losses in the stator and rotor) of I_{2l} may be calculated by the relation

$$FH = \frac{V_1^2}{V_{1t}^2}(FH_t) \text{ (approximately)}$$

where

$$FH_t = \frac{\text{Phase power input} - \text{phase core loss at } V_{1t}}{V_{1t}}$$

The in-phase component of the current $(FH+HI)$ and the current I_{1l} enable one to calculate the angle θ_{1l}.

Resistance measurements.—The equivalent resistance of the windings referred to the stator may be obtained from the impedance test (see Article 48).

To predetermine the performance curves of a motor from its circle diagram it is necessary to know the relative values of the stator and rotor resistances. In a squirrel-cage motor the resistances of the rotor cannot be directly measured, but it is possible to determine the value of the d-c. resistance r'_1 of the stator winding and assume that the effective resistance is $r_1 = 1.25r'_1$ approximately. The total equivalent resistance determined by the impedance test may be expressed by the relation

$$R_1 = 1.25r'_1 + r_{21}$$

Naturally the factor by which r'_1 must be multiplied to obtain the effective resistance depends on the design of the motor and the assumed factor 1.25 must be used with caution.

When the motor has a wound rotor, both the stator and rotor winding resistances may be measured by direct current. A good approximation for the effective resistance of each winding may be obtained by comparing the value of d-c. resistance referred to the stator winding with the equivalent resistance obtained by the impedance test. Thus

$$R_1 = c(r'_1 + r'_{21})$$

$$r_1 = cr'_1$$

and

$$r_{21} = cr'_{21} \quad \text{(approximately)}$$

It is here assumed that the stator and rotor are wound for the same number of phases and that the factor c is the same for both windings.

153. Construction of the Circle Diagram from Test Values (see Fig. 122).—The circle diagram is constructed as follows:

1. Locate $\dot{V}_1$ the phase voltage.
2. Locate I_N. (The value of I_N and θ_N are determined from the no-load test.)
3. Locate I_{1l}. (The value of I_{1l} and θ_{1l} are obtained from the impedance test.)
4. Draw OM 90 degrees displaced from V_1.
5. Draw AG parallel to OM.
6. Draw the line AF.
7. Draw perpendicular bisector of AF. This line cuts the line AG and the point of intersection determines the center of the circle.

8. Draw a line from A to K. Make the distance HK such that $(HK)V_1$ = copper loss in a stator phase due to the current I_{21l}.

154. Determination of the Performance Curves of an Induction Motor by the Circle Diagram.—The efficiency, slip, torque, and power factor for a given output are readily determined from the circle diagram. (Fig. 123 is similar to Fig. 122, but some construction lines are added to simplify several geometrical proofs.)

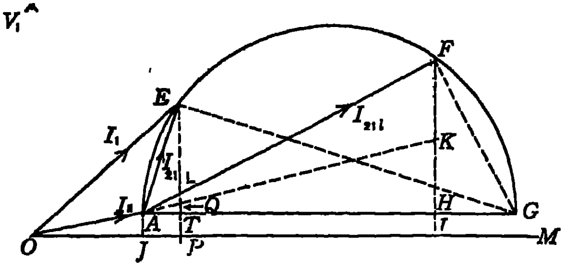


FIG. 123.—Circle Diagram of an Induction Motor

Phase power input.—For a given load current I_1 the phase power input is equal to the product of the phase voltage and the in-phase component of the phase current. Thus

$$\text{Phase power} = V_1(EP) \tag{3}$$

Phase power losses.—The phase power due to friction, windage, core loss, and copper loss due to the exciting current may be calculated by finding the product $V_1(AJ)$. This portion of the total power loss in the motor is assumed constant; i.e., the components AJ, TP, and HI are assumed equal. The assumption just made is not strictly true, for as the slip of the motor increases, the friction and windage losses are not likely to decrease by the same amount that the core losses increase.

The equivalent phase copper loss of the motor for rated impressed voltage and rotor locked conditions = $V_1(FH)$. It will now be demonstrated that for a given current I_{21} the phase copper loss = $V_1(LT)$.

The triangles ATL and AHF are similar. Therefore

$$\frac{LT}{AT} = \frac{FH}{AH} \tag{A}$$

The triangles AHF and AFG are similar. Therefore

$$AH=\frac{(AF)^2}{AG}=\frac{I_{21l}^2}{AG} \qquad \text{(B)}$$

The triangles ATE and AEG are similar. Therefore

$$AT=\frac{(AE)^2}{AG}=\frac{I_{21}^2}{AG} \qquad \text{(C)}$$

From A, B, and C it follows that

$$LT=I_{21}^2\frac{FH}{I_{21l}^2} \qquad \text{(D)}$$

If

$$FH=\frac{R_1I_{21l}^2}{V_1}$$

it follows that

$$LT=\frac{R_1I_{21}^2}{V_1} \qquad \text{(E)}$$

The total phase power loss for a given input current I_1 is equal to the sum of the excitation losses and the copper losses. Therefore

$$\text{Phase power losses}=V_1(LP) \qquad (4)$$

Phase power output.—

$$\text{Output}=\text{Input}-\text{losses}$$

$$\text{Phase output}=(EP-LP)V_1=(EL)V_1 \qquad (5)$$

Efficiency.—

$$\xi=\frac{\text{Output}}{\text{Input}}=\frac{V_1(EL)}{V_1(EP)}100=\frac{EL}{EP}100 \qquad (6)$$

Slip.—The rotor phase copper loss

$$r_{21}I_{21}^2=s(E_{21}I_{21}\cos\theta_{21})$$

Therefore

$$s=\frac{r_{21}I_{21}^2}{E_{21}I_{21}\cos\theta_{21}} \qquad (7)$$

The slip may be determined from the circle diagram as follows:

The triangles AQL and AKF are similar. Therefore

$$\frac{LQ}{AQ}=\frac{FK}{AK} \qquad \text{(F)}$$

The triangles ATQ and AHK are similar. Therefore

$$\frac{AQ}{AT}=\frac{AK}{AH} \qquad \text{(G)}$$

From F and G it follows that

$$\frac{LQ}{FK}=\frac{AT}{AH} \tag{H}$$

but

$$\frac{AT}{AH}=\frac{I_{21}^2}{I_{21l}^2}$$

Therefore

$$LQ=\frac{I_{21}^2}{I_{21l}^2}FK$$

If

$$V_1(KH)=r_1I_{21l}^2$$

then

$$V_1(FK)=r_{21}I_{21l}^2$$

and

$$V_1(LQ)=r_{21}I_{21}^2 \tag{I}$$

The total electrical input to a phase of the rotor = output + rotor copper loss. Therefore from equations (5) and (I) it follows that

$$E_{21}I_{21}\cos\theta_{21}=V_1(EQ) \tag{J}$$

and from equations (I), (J), and (7) it follows that

$$s=\frac{LQ}{EQ} \tag{8}$$

Phase torque.—The total input to the rotor is

$$\frac{T(2\pi n_1)746}{550}=qV_1(EQ)+(\text{friction}+\text{windage}+\text{rotor core loss})$$

and the useful torque per phase

$$T_{\text{phase}}=\frac{T}{q}=\frac{550}{746}\,\frac{V_1(EQ)}{2\pi n_1}$$

Power factor.—The angle between V_1 and I_1 is readily measured and the corresponding power factor calculated.

Maximum power factor.—The maximum power factor is obtained when the current I_1 is tangent to the circle.

Maximum power.—The current for maximum power is obtained by drawing a line parallel to AF and tangent to the circle. The point of tangency determines the location of point E.

Maximum torque.—The current input for maximum torque is obtained by drawing a line parallel to AK and tangent to the circle. The point of tangency locates the point E.

155. Measurements Made on an Induction Motor.—

Rating.—1000-horse power· 2200-volt; 25-cycle.

Temperature.—75° C.

No-load test.—

Rotor running free
Terminal voltage 2200
Frequency 25 cycles
No-load current 43 amperes
No-load power 24.7 kw. (three phases)

Impedance test.—

Rotor short-circuited and locked.
Terminal voltage 1000
Frequency 25 cycles
Terminal current 837 amperes
Total power input 418.5 kw. (three phases)

Resistance test.—

Y-phase d-c. resistance of a stator phase = 0.79 ohm.
Y-phase d-c. resistance of a rotor phase = 0.34 ohm.
Ratio of primary turns to secondary turns = 1.59

PROBLEMS

1. A receiving circuit having a constant resistance of 10 ohms and a variable inductive reactance is connected to a 110-volt, 60-cycle source of power. Construct voltage drop and current circle diagrams for this circuit.

2. A receiving circuit having a constant inductive reactance of 10 ohms and a variable resistance is connected to a 110-volt, 60-cycle source of power. Construct voltage drop and current circle diagrams for this circuit.

3. Construct the current circle diagram for the induction motor specified in Problem 7, Chapter XIII. (Assume that the rotor is short-circuited). From the circle diagram predetermine the following performance curves:

(*a*) Torque plotted against slip.
(*b*) Efficiency plotted against h.p. output.
(*c*) Power factor plotted against h.p. output.

4. Construct the current circle diagram for the induction motor specified in Problem 7, Chapter XIII. (Assume that resistances are connected in each rotor phase so as to obtain the maximum torque at starting.) Determine a set of curves similar to those obtained in Problem 3.

5. Compare the results obtained in Problems 3 and 4 with those obtained by the analytical method. (See your solution to Problem 7, Chapter XIII.)

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CHAPTER XV

STARTING; SPEED CONTROL; AND POWER FACTOR CORRECTION OF POLYPHASE INDUCTION MOTORS

156. Starting of Polyphase Squirrel-cage Induction Motors.—Small squirrel-cage motors may be started by connecting the primary winding directly across full line voltage. The current required during the starting period is decidedly abnormal, but the heat developed by this current is small and no injury is done to the windings of the motor or to the circuit supplying the motor.

Large squirrel-cage motors, if started by connecting directly across the line, take from 5 to 10 times normal rated current and these low-power factor currents cause an overload on the distributing system and disturb every electrical load connected to the system. For this reason induction motors are frequently started at reduced voltage. A very efficient method of reducing the voltage is to use an auto-transformer. A special double throw switch is required to make the connection from the auto-transformer to the motor. One position of the switch (the starting position) connects the motor to approximately 60 per cent rated line voltage and the other position to full line voltage. The actual method of starting consists in throwing the switch in the starting position and allowing it to remain there until the rotor of the motor has gained considerable momentum. The switch is then thrown to the running position.

157. Starting of a Wound Rotor Polyphase Induction Motor.—Motors having wound rotors may be started by inserting resistance in the rotor winding circuit and connecting the stator winding directly across the line. Equation (22) in Chapter XIII shows that the maximum torque developed by the motor is independent of the rotor resistance provided the rotor resistance referred to the stator is not greater than that calculated by equation (21) for a slip of unity. On the other hand the slip at which the maximum torque occurs depends on the rotor resistance. Thus

by inserting the proper resistance in the rotor a high to
be obtained during the starting period. The resistance m
starting is well suited for motors driving mechanical loads
require high torques at low speeds. Since an induction motor is electrically equivalent to a static transformer it is apparent that the insertion of the proper resistance in the rotor prevents excessive primary currents and low primary power factors.

158. Common Methods of Speed Control.—Most of the methods used for controlling the speed of induction motors may be studied by referring to the following equations:

$$n_2 = (1-s)n_1 = (1-s)\frac{2f_1}{p_1} \tag{1}$$

and

$$T = \frac{550}{746}\,\frac{qV_1^2}{2\pi n_1}\left[\frac{sr_{21}}{(r_1s+r_{21})^2+s^2(x_1+x_{21})^2}\right] \tag{2}$$

The first equation is used for calculating the rotor speed and the second one for calculating the torque for a given slip. (See Chapter XIII relative to derivations of these equations.) The equations show that for a given torque the speed of a given induction motor is determined by the following factors:

(*a*) Stator frequency f_1.
(*b*) Stator voltage V_1.
(*c*) Number of poles p_1.
(*d*) Stator and rotor constants x_1, x_{21}, r_1, and r_{21}.

Changing the impressed frequency.—This method of speed control is limited to applications where the total load on the prime mover consists of induction motors requiring similar speed variations at the same time. Electrically propelled ships are sometimes controlled by this method. The chief objection to the method is that a given prime mover operates efficiently over only a limited range of speed.

Changing the impressed voltage.—This method of speed control is based on the fact that the torque for a given slip depends on the square of the impressed voltage. This method has the objectionable feature in that the break-down torque of an induction motor depends on the square of the impressed voltage. A too great reduction in voltage may result in a very low breakdown torque.

Changing the number of poles.—In this method of speed control special windings are required on the motor. The number of poles may be changed by changing the connections of the coils of a given winding or by using separate windings. In small motors the connections of the windings may be changed by a special switch, but large motors require separate windings. For maximum torque a change in the number of poles of the stator requires a similar change in the number of poles of the rotor. That is, wound rotor motors having two separate windings on the stator require two separate windings on the rotor. In squirrel-cage motors the number of poles on the rotor are determined by the number of poles on the stator and only one winding is required on the rotor.

Changing the stator and the rotor constants.—The torque developed by an induction motor for a given slip is a function of the constants of the stator and the rotor circuits. An increase in the inductive reactance of either the stator or the rotor windings results in a greater slip, but the power factor of the motor and its maximum torque are decreased. The addition of resistance to the stator winding increases the slip, but the rI^2 losses are increased and the maximum torque is decreased.

One of the best methods for controlling the speed of an induction motor is to insert a variable resistance in the rotor circuit. Within certain limits [see equation (23), Chapter XIII] the rotor resistance does not change the maximum torque. The chief objection to this method of speed control is that the rI^2 losses are increased.

159. Speed Control of Induction Motors by Concatenation.—Several efficient running speeds may be obtained by connecting two induction motors in concatenation (see Fig. 124). The rotors of the two motors are mounted on the same shaft or are mechanically connected together by some other method. The stator winding of one of the motors is supplied by current from the rotor winding of the other motor. The actual electrical connection of the motors depends on the speed desired. The motors are said to be connected in direct concatenation when they produce starting torques in the same

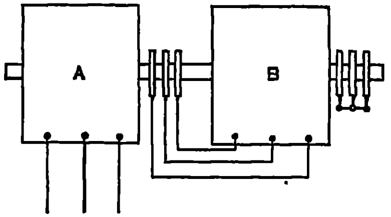


Fig. 124.—Induction Motors Connected in Concatenation

direction and in differential concatenation when they produce starting torques in opposite directions.

If the two motors are coupled to the same shaft and are electrically connected in direct concatenation the speed of the set may be calculated as follows:

Let s_1 = slip of motor A;
s_2 = slip of motor B;
f_1 = frequency impressed on stator of motor A;
f_2 = frequency impressed on stator of motor B;
p_1 = number of poles of motor A;
and
p_2 = number of poles of motor B.

The frequency of the voltage impressed on the stator winding of motor B is equal to the slip frequency (s_1f_1) of motor A. Both motors must operate at the same speed and therefore

$$(1-s_1)\frac{2f_1}{p_1}=(1-s_2)\frac{2s_1f_1}{p_2} \tag{3}$$

or

$$s_1=\frac{p_2}{p_1-s_2p_2+p_2} \tag{4}$$

When the rotor of motor B is short-circuited the conditions for high power factor and high torque are best when the slip s_2 is small and if we assume s_2 very small, the slip of motor A becomes

$$s_1=\frac{p_2}{p_1+p_2} \quad \text{(approximately)} \tag{5}$$

and the synchronous speed of the set becomes

$$n_s=\left(1-\frac{p_2}{p_1+p_2}\right)\frac{2f_1}{p_1}=\frac{2f_1}{p_1+p_2} \tag{6}$$

Note that in direct concatenation the synchronous speed of the set is independent of which one of the motors is connected to the supply.

Several other efficient running speeds are possible. By connecting motor A to the supply with its rotor short-circuited the synchronous speed of the set becomes $n_s=\frac{2f_1}{p_1}$. Similarly by connecting B this way the synchronous speed becomes $n_s=\frac{2f_1}{p_2}$

A high speed may be obtained by connecting the motor in differential concatenation. This is done by interchanging two of the stator leads of motor *B*. When connected this way the motors produce starting torques in opposite directions and the direction of rotation of the set is determined by the motor which produces the highest torque at starting. The torque developed by a combination like this varies considerably throughout the speed range of the set. The torque is zero when the motor which is fed from the supply is driven at synchronous speed. For example, when a 6-pole motor and a 4-pole motor are connected in differential concatenation and the stator of the 6-pole motor is fed from a 60-cycle supply the torque developed by the set is zero when the speed is $\frac{2\times 60}{6}=20$ r.p.s. This must be so because at this speed no e.m.f. and current are generated in the rotor of the 6-pole motor. At speeds above 20 r.p.s. the 4-pole motor feeds the rotor of the 6-pole motor with reverse phase rotational currents making a high speed possible. The transition from a speed below 20 r.p.s. to a speed above 20 r.p.s. may be made by starting the set in the proper direction using the 4-pole motor alone and then switching back to the 6-pole motor and making the proper interconnections at the same time. The limiting value of the speed obtainable by this method is $\frac{2f_1}{p_1-p_2}=\frac{2\times 60}{6-4}=60$ r.p.s.

This method of speed control is used where several motors are required to do similar work. Electric locomotives when equipped with polyphase induction motors are controlled by this method. Induction motors in steel mills are sometimes operated in concatenation.

160. Power Factor Correction of Induction Motors.—The electrical circuits of an induction motor are equivalent to those in a static transformer. For this reason the power factor of the primary circuit of the motor depends largely on the power factor of its secondary circuit. In an ordinary induction motor equipped with no power factor correcting devices the power factor of the secondary winding is always less than unity. Since the primary winding is always inductive and the secondary circuit referred to primary turns is inductive it follows that an ordinary induction motor must operate at lagging power factor regardless of the load on the motor.

The best way to improve the power factor of a given inducti motor is to change the phase relations between the secondary e.m.f. and the secondary current. This may be done by injecting an e.m.f. of slip frequency from a separate machine into the rotor circuit or by connecting a condensive impedance in series with the rotor circuit. Several methods for correcting power factor and for controlling speed are described in the following articles.

161. Phase Advancer.—The chief purpose of the phase advancer is to improve the power factor of the induction motor to which it is connected. What it actually does is to advance the phase of the current in the rotor relatively to the e.m.f. which produces the current. A leading current in the rotor changes the direction of the rotor magnetic reaction and the magnetizing action of the rotor currents reduces the magnetizing current required in the primary winding of the induction motor. By sufficiently advancing the phase of the rotor current it is possible to supply all the magnetizing current through the rotor winding. The magnetizing current of an induction motor on account of the air-gap is a relatively large low-power factor current and its elimination from the primary circuit must result in a better power factor.

One type of phase advancer intended for connection to a two-phase rotor is shown in Fig. 125*a*. The machine has an armature and a commutator which are similar in every way to those on a d-c. generator. The stator is made of laminated iron, but no stator winding is required. In a two-phase two-pole machine four brush sets located 90 electrical space degrees apart are necessary. In actual operation the armature of the phase advancer is electrically connected to the slip rings of the motor and the rotor of the phase advancer is driven by an auxiliary variable speed motor.

The frequency of the currents circulating through the armature of a phase advancer must equal the slip frequency of the current in the rotor of the induction motor to which it is connected. A revolving magnetic field is produced about the armature whose speed relative to the mechanically fixed brushes is equal to $sf_1\left(\frac{2}{p}\right)$ r.p.s. The speed of this field does not depend on the speed of the armature due to the fact that the equivalent mesh between the brushes never changes.

The complete action of the phase advancer may be described as follows: Referring to Fig. 125*a* when the current entering brush 3 has a maximum value the revolving field is located at the position indicated in the figure. At this instant the e.m.f. due to the rotation of the flux and the armature (the rotational e.m.f.) is zero between brushes 3 and 4 and has a maximum value between brushes 7 and 8. This is equivalent to saying that the rotational e.m.f. between brushes 7 and 8 is in time phase or in time phase opposition with the current through the brushes 3 and 4. Similarly the rotational e.m.f. between brushes 3 and 4 is in time phase or in time phase opposition with the current through brushes 7 and 8. Now since the currents 3–4 and 7–8 are 90 degrees apart it follows that the rotational e.m.f. from 3–4 and the current from 3–4 are displaced by the same angle. The actual direction of the rotational e.m.f. in a phase depends on the speed and the direction of rotation of the armature it may lead or lag the current in the phase. The frequency of this e.m.f. is always equal to the frequency of the e.m.f. in the rotor winding of the induction motor.

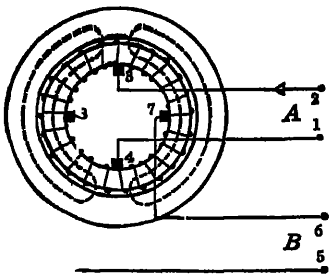


FIG. 125*a*.—Rotary Phase Advancer

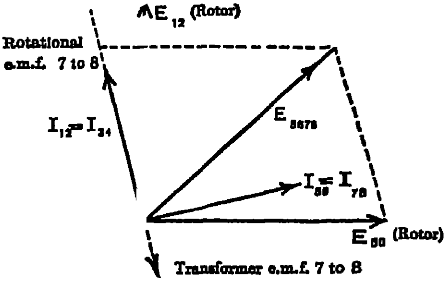


FIG. 125*b*.—Vector Diagram for a Rotary Phase Advancer

Since there is no secondary winding on a phase advancer and since the reluctance of the magnetic circuit is high compared to that of a power transformer it follows that the transformer e.m.f. in a phase of the phase advancer lags the current in the phase by nearly 90 degrees. On this basis the transformer e.m.f. and the rotational e.m.f. in a given phase are in phase or in phase opposition, depending on the direction of the rotational e.m.f. The magnitude of the transformer e.m.f. depends on the current and the frequency s_1f_1 while the rotational e.m.f. depends on the current, the frequency s_1f_1, and the speed of the armature of the phase advancer.

The resultant e.m.f. in a phase such as 5, 6–7, 8 is E_{56} (*rotor e.m.f.*)$+E_{78}$(*transformer e.m.f.*)$+E_{78}$(*rotational e.m.f.*). The current $I_{5,\,6-7,\,8}$ lags the resultant e.m.f. but may be made to lead the rotor e.m.f. A vector diagram showing the voltage and current relations is given in Fig. 125*b*.

162. Speed Control and Power Factor Correction by the Use of a Shunt Wound Polyphase Commutator Motor.—Fig. 126 illustrates a method of speed control and power factor correction in which the electrical power developed in the rotor winding of an induction motor is utilized by a shunt wound commutator motor.

The shunt commutator motor has a rotor which resembles that of the armature and commutator of an ordinary d-c. motor. The number of brush sets required is determined by the number of poles and the number of phases. A two-pole, three-phase motor requires three brush sets. The stator winding is similar to that on the stator of an ordinary polyphase motor and is fed with power through an auto-transformer. The rotor winding is fed by connecting the brushes directly across the supply. Two revolving magnetomotive forces are produced, one due to the stator currents and one due to the rotor currents. The speed of these m.m.f.'s relatively to the mechanically fixed brushes are the same and since the axes of the m.m.f.'s do not coincide a torque is produced.

If the shunt motor is connected to the rotor of an induction motor as shown in Fig. 126 the magnitude and the phase of the counter electromotive force of the commutator motor determine the slip and power factor of the induction motor. The counter e.m.f. may be controlled by changing the ratio of transformation of the auto-transformer through which the field winding is supplied and the phase of the counter e.m.f. is determined by the position of the brushes.

For a given ratio of the auto-transformer the field flux of the commutator motor has a constant r.m.s. value regardless of the speed of the induction motor. That this must be so is evident when we keep in mind that the reactance of the auto-transformer and the reactance of the field winding of the commutator motor both increase at the same rate as the rotor voltage of the induction motor increases. The counter voltage developed by the commutator motor depends on the field flux and on the relative speed between its field and the rotor. Since the relative speed between the revolving field and the rotor of the commutator motor cannot change appreciably (a small change represents a large power) and since the field flux of the commutator motor is constant it follows that for a given ratio of the auto-transformer the counter voltage of the commutator motor is nearly constant regardless of the mechanical load on the induction motor.

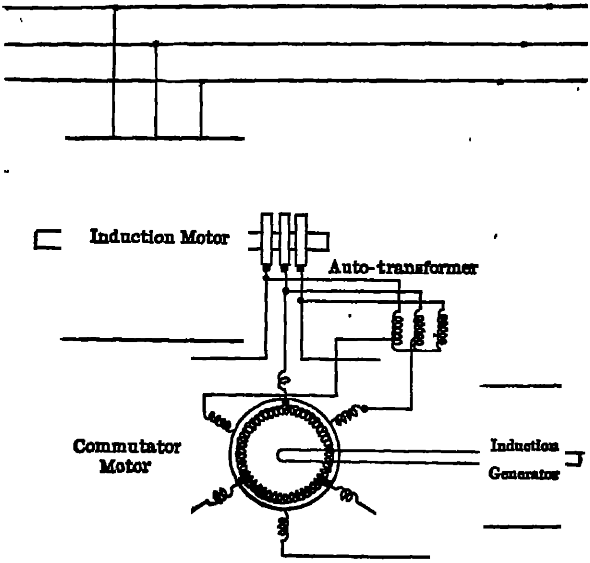


Fig. 126.—Speed Control and Power-factor Correction of an Induction Motor

Since the leakage impedances of the rotors of the induction motor and the commutator motor are small it follows that the slip voltage of the induction motor must be approximately equal to the counter voltage of the commutator motor and consequently must be nearly constant. Therefore, for a given setting of the auto-transformer the slip of the induction motor cannot change appreciably as the load on the motor is changed; i.e., the speed regulation of the induction motor is fairly good.

163. Double-range Speed-regulating Equipment for Induction Motors.—This method of speed control involves the use of a polyphase commutator motor and an ohmic-drop exciter. By this method speeds above as well as below synchronism are readily obtained. This method, due to the cost of the auxiliary apparatus required, is applicable to large machines only. The apparatus required is shown in Fig. 127.

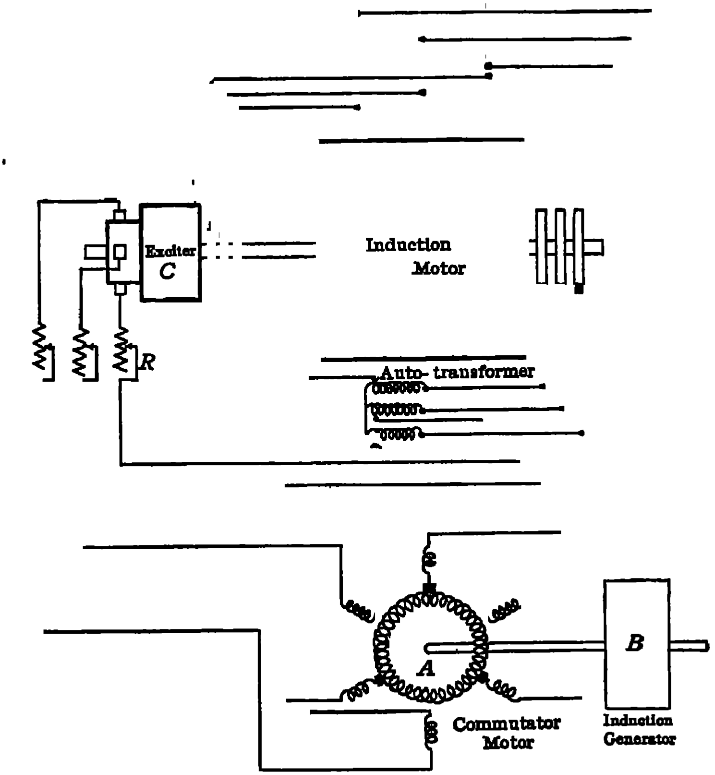


FIG. 127.—Double Range Speed Regulating Set for an Induction Motor

Machine *A* is a polyphase commutator motor which under certain conditions operates as a generator. Machine *B* is an induction generator mounted on the same shaft as *A*. Under certain conditions the latter machine operates as a motor. As far as machines *A* and *B* are concerned this method of speed control is similar to that in Fig. 126.

In order to operate at synchronism and above synchronism machine *C* (the ohmic-drop exciter) is required. The armature of the exciter is similar to the armature in a synchronous converter, but by extending the armature laminations beyond the armature conductors it is possible to omit the stator which is always required in a converter. Since this machine is mechanically driven by the induction motor and since the collector-rings are connected directly across the supply it follows that the frequency of the voltage obtained across the brushes bearing on the commutator is equal to the frequency of the voltage induced in the rotor of the induction motor.

The speed of the induction motor is controlled by changing the ratio of transformation of the auto-transformer. At synchronous speed the excitation of the field of the commutator motor is obtained entirely from the ohmic-drop exciter which at this speed delivers direct-current. The transition from synchronism to speeds above synchronism is obtained by reversing the phase rotation of the currents supplied to the rotor of the induction motor. This is done by sufficiently decreasing the resistance *R*.

PROBLEMS

1. Calculate the approximate value of the starting current when the motor specified in Problem 7—Chapter XIII is connected directly across full line voltage (rotor short-circuited).

2. Repeat the calculations in Problem 1 when the rotor resistance is such as to obtain the maximum torque at starting.

3. Two induction motors are mounted on the same shaft and are operated in concatenation. One of the motors has 8 poles and the other 6. The frequency of the power source is 60 cycles per second. Calculate the approximate running speeds possible.

4. A 20-horse power, 4-pole, 220-volt, 60-cycle induction motor and a 20-horse power, 6-pole, 220-volt, 60-cycle induction motor have a common shaft. The stator of the 4-pole motor is connected to a 220-volt, 60-cycle supply. The rotor of the 4-pole motor is connected directly to the stator of the 6-pole motor. The rotor of the 6-pole motor is short-circuited. Both motors have a turn ratio of unity. (*a*) Calculate the maximum safe continuous output of the motors. (*b*) What proportion of the load does each motor take?

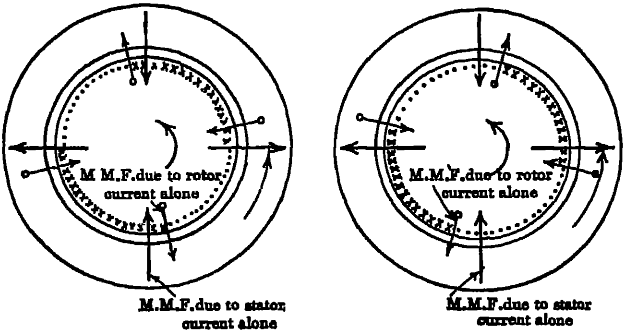
5. Referring to the motors specified in Problem 6, show that the insertion of the right amount of resistance in the rotor circuit of the 6-pole motor results in a higher starting torque in both motors.

CHAPTER XVI

INDUCTION GENERATOR; PHASE CONVERTER; SYNCHRONOUS-INDUCTION MOTOR; AND FREQUENCY CHANGERS

164. Induction Generator.—An ordinary induction motor may be used as a generator provided its rotor is driven above synchronous speed in the direction of rotation of the air-gap flux and provided a condensive load is connected across its terminals. The initial flux in the air-gap must be obtained by exciting the stator winding from a source of a-c. power.

It is apparent that when the rotor is driven above synchronous speed (negative slip) the direction of the currents in the rotor are reversed with respect to the directions of these currents when the machine is operated as a motor (see Figs. 128*a* and 128*b*). The direction of the torque is also reversed and mechanical power must now be supplied to the rotor to keep the machine going. Power is now transferred from the rotor to the stator and the component of the phase current which represents power is reversed.

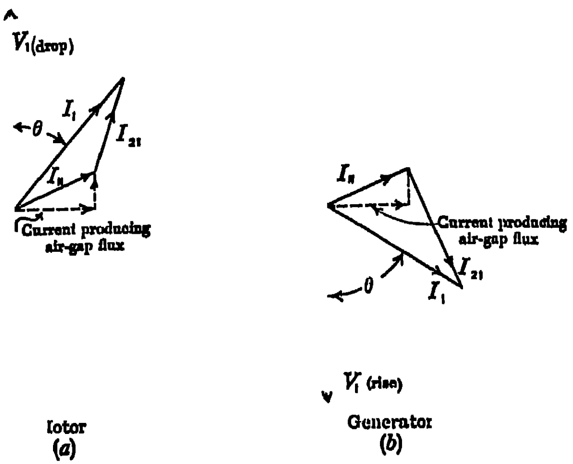

M M.F.due to rotor current alone
M.M.F.due to stator current alone
M.M.F.due to rotor current alone
M.M.F.due to stator current alone

Motor
(*a*)

Generator
(*b*)

FIG. 128.—Magnetomotive Forces Due to the Currents in the Windings of an Induction Motor and Induction Generator

It should be noted that the direction of the stator field is not reversed and therefore it may be assumed that the magnetizing component of the current which produces the air-gap flux is the same in the motor and the generator. Complete vector diagrams showing the relations between the phase voltages and phase currents are given in Figs. 129*a* and 129*b*.



Motor
(*a*)

Generator
(*b*)

FIG. 129.—Vector Diagrams for an Inductive Motor and Induction Generator

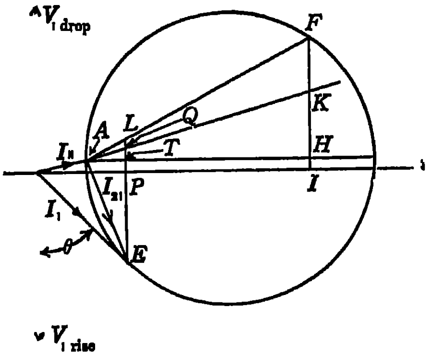


FIG. 130.—Current Circle Diagram for an Induction Generator

The complete performance of an induction generator may be calculated analytically by substituting negative values of slip in the formulas which were derived for the induction motor. The performance may also be predicted from the circle diagram (Fig. 130) which is obtained from the circle diagram of the induction motor by merely completing the circle.

The location of the lines *AF* and *AK* remain unchanged. The line *LT* is proportional to the square of the line *AE* and is erefore a function of the copper loss in a phase of the generator. he line TP is proportional to the friction windage and core loss d the line PE is proportional to the output of the generator.

The losses, output, efficiency and slip may be calculated as llows:

$$\text{Rotor copper loss} = q(LQ)V_1$$

$$\text{Stator copper loss} = q(QT)V_1$$

$$\text{Core loss friction and windage} = q(TP)V_1$$

$$\text{Output} = q(PE)V_1$$

$$\text{Efficiency} = \frac{PE}{LE}100$$

$$\text{Slip} = \frac{LQ}{QE}100$$

Induction generators are usually operated in parallel with ynchronous generators. When once in operation the synchronous enerators may be disconnected from the induction generator rovided the load connected to the induction generator is condensive. Synchronous converters and synchronous motors may e supplied by individual induction generators. Electric braking y induction generator action may be accomplished on locomotive quipped with induction motors.

The advantages of an induction generator are:

1. It need not be synchronized.
2. The short-circuit current is small.

(A short-circuit immediately destroys the field flux.)

3. The efficiency is fairly high.

The disadvantages of an induction generator are:

1. The power factor at which it operates must be leading nd is determined by the constants of the machine and the driving orque of its prime mover.
2. The initial excitation must be furnished by a synchronous nachine.
3. The exciting volt-amperes are much greater than in an rdinary synchronous generator.

165. Phase Converters.—The phase converter is a revolving nachine which is used in transforming from polyphase current o single-phase current or from single-phase current to polyphase urrent.

Polyphase to single-phase conversion.—It is well known that a two-phase induction motor or synchronous motor after once started will run on one phase. The disconnected phase of the motor may be used as a source of single-phase power.

In order to convert from two-phase to single-phase the connection shown in Fig. 131 is used. Phase *A* of the polyphase motor is connected to one phase of the supply and phase *B* is connected in series with the load to the other phase.

The connections are so made that the voltage across the load is approximately double the phase voltage impressed on the converter. The converter is put into operation as an ordinary polyphase motor and after it is up to speed the connections to phase *B* are reversed so that the voltage generated in this phase is added to the impressed voltage of this phase. In actual operation phase *A* acts as a receiver of power and phase *B* as a generator. Each phase of the converter operates at approximately the same voltage and at approximately the same current. The volt-ampere capacity of the converter must be equal to the volt-amperes of the load. By this method a single-phase load may be drawn from a two-phase system without unbalancing the load on the system.

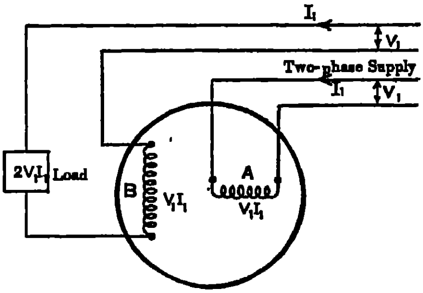
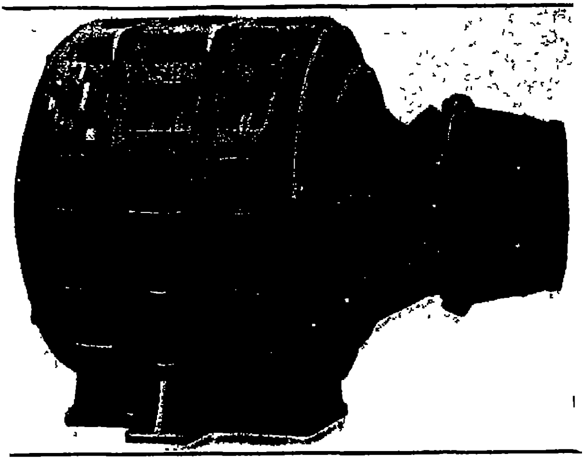


Fig. 131.—Phase Converter (Two-phase to Single-phase)

Single-phase to polyphase conversion.—The Norfolk and Western locomotives are driven by three-phase induction motors. Power is supplied to the locomotives from a single-phase source by an over-head trolley. The connecting link between the single-phase source and the three-phase motors is a phase converter (see Fig. 132*a*).

The connections of the converter to the induction motors and to the trolley are shown in Fig. 132*b*. The converter is driven by power supplied to phase *A*. Phase *B* corresponds to the teaser coil of a transformer T-connection. The converter is brought up to speed by a single-phase commutator motor.



(*By the courtesy of the Westinghouse Co.*)

FIG. 132*a*.—Phase Converter (Single-phase to Three-phase)

166. Synchronous-Induction Motor.—This motor operates below or at synchronous speed depending on the mechanical output of the motor. Below synchronous speed the performance of the motor is identical with that of an ordinary polyphase induction motor. At synchronous speed its performance is similar to that of a synchronous motor. Below synchronous speed the power factor is determined by the constants of the circuits and at synchronous speed the power factor may be adjusted by changing the field excitation.

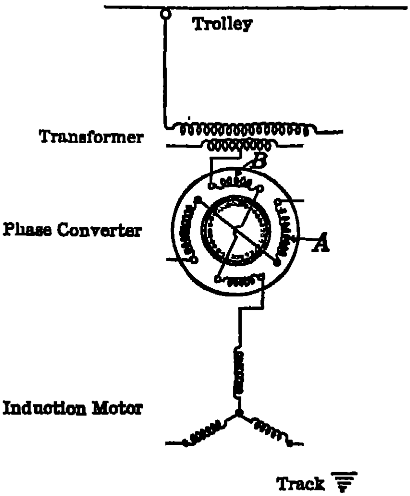
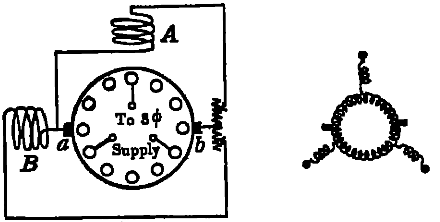


FIG. 132*b*.—Diagram of Connections of a Phase Converter

A diagram showing the circuits of the motor is given in Figs. 133*a* and 133*b*. The power is supplied to the rotor instead of the stator as in the ordinary motor. The rotor winding is so constructed that a relatively high a-c. voltage may be applied to the rings and yet the voltage across the brushes on the commutator is small. A special star-mesh armature connection shown in Fig. 133*b* is required for this purpose.

The operation of the motor may be explained as follows: Below synchronous speed the polyphase current in the armature winding produces a revolving magnetic field which cuts the two stator windings *A* and *B* and induces e.m.f.'s of slip frequency in them. An e.m.f. of slip frequency is also induced between brushes *a* and *b* due to the cutting of the air-gap flux. The two e.m.f.'s acting on the circuit formed by coils *A* and *B* produce a current which is also of slip frequency. A magnetic reaction between the stator and the rotor circuits takes place and torque is produced as in an ordinary polyphase induction motor.



(*a*) (*b*)

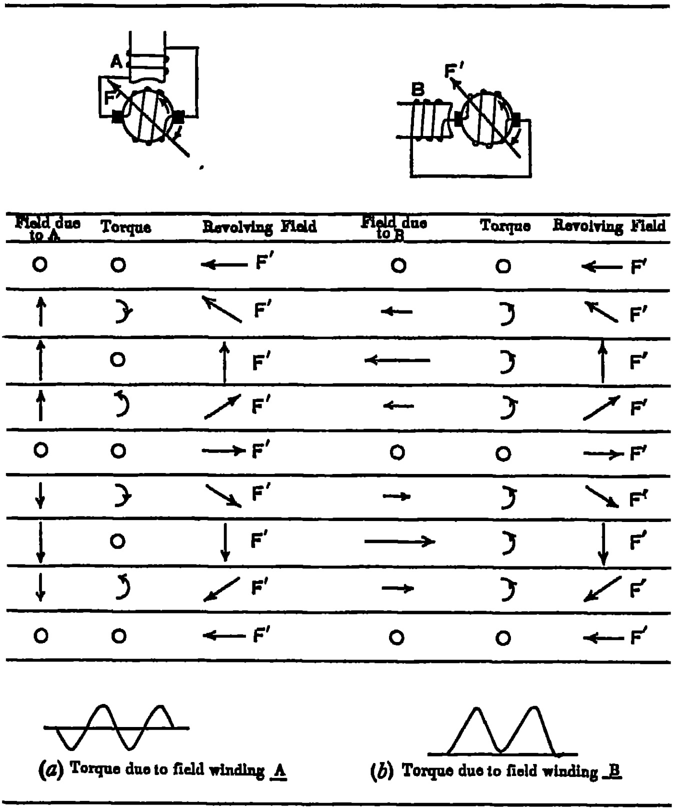
FIG. 133.—Wiring Diagrams of a Synchronous Induction Motor

As the machine speeds up the cutting e.m.f. impressed on windings *A* and *B* becomes greater than that obtained by induction and the current in these windings is set up primarily by the cutting e.m.f. When synchronism is approached a high pulsating unidirectional synchronizing torque is produced by winding *B* and the rotor is pulled into synchronism. Winding *A* produces an alternating synchronizing torque (see Figs. 134*a* and 134*b*).

When synchronism is actually obtained the stator windings are supplied by direct-current and the machine will operate as a synchronous motor. Winding *A* may now be considered as producing the torque field and winding *B* a compensating field.

167. Frequency Changers.—Frequently it is desirable to transfer power from one a-c. circuit to another circuit operating at a different frequency. The machine which forms the connecting link between the two circuits is called a frequency changer. Motor-generator sets are always used for this purpose. Both synchronous and asynchronous machines may be used.

Synchronous—synchronous frequency changer.—This type of frequency changer consists of a synchronous motor driving a synchronous generator. If a-c. power at 25 cycles is to be changed to 60-cycle power it is necessary that the motor is wound for 10 poles and the generator for 24 poles. The operating speed of the set is 300 r.p.m. and this is the maximum speed at which the transformation can be made for the number of poles specified represent the minimum number.



(*a*) Torque due to field winding A (*b*) Torque due to field winding B

FIG. 134.—Synchronizing Torques Produced in a Synchronous-Induction Motor

The synchronous machines must be synchronized to their respective circuits. This is difficult to accomplish for when the motor is in synchronism it is not likely that the generator is also in synchronism. Exact synchronism is obtained by "slipping poles" (see Article 133 on the synchronous converter) and by axially displacing the stator of one of the machines with respect to the other (the stator of at least one of the machines is mounted on a cradle). By this method power may be transferred from the low-frequency system to the high-frequency system or vice versa. The reversal of power is accomplished by changing the phase angle between the excitation e.m.f.'s of the machines. This may be done by axially displacing one of the stators.

Induction-type frequency changer.—This type of frequency changer consists of a wound rotor induction machine (motor) driven by a synchronous motor. The stator of the induction machine is connected to one power system and the rotor to another power system which operates at a different frequency. When a frequency f_1 is applied to the stator of the induction machine and the rotor is driven in the opposite direction to that of the revolving field produced by the stator currents the frequency of the current in the rotor circuit is $f_2 = \left(\frac{n_1 + n_2}{n_1}\right) f_1$, where n_1 represents the synchronous speed of the stator field and n_2 represents the speed of the rotor. The chief objection to this method of changing frequency is that a poor voltage regulation on one power system affects the voltage of the other system.

PROBLEMS

1. An induction generator and a synchronous generator are operated in parallel to supply a synchronous converter. (*a*) What will happen if the synchronous generator is disconnected? (*b*) What factors determine the frequency and the terminal voltage in (*a*)?

2. The induction motor specified in Problem 7—Chapter XIII is operated as an induction generator. (*a*) Construct the current circle diagram for the generator. (*b*) Predetermine the following performance curves:
   (1) Power output plotted against values of slip.
   (2) Efficiency plotted against power output.
   (3) Power factor plotted against power output.

3. A phase converter is connected to a load as shown in Fig. 132. The power input to the three-phase load is 425 kv-a., at 440-volts, and 87 per cent lagging power factor. (*a*) Calculate the power in the two windings of the phase converter. (Assume that the phase converter is perfect; that is, it has no losses and no leakage reactance.) (*b*) How much power is supplied directly to the load without going through the phase converter. (*c*) Construct a voltage-current vector diagram showing the voltage and current relations in the load and the converter.

4. An induction frequency changer is used to transform 10,000 kw. from a 25-cycle to a 60-cycle circuit. The stator of the frequency changer is wound for 14 poles. (*a*) Calculate the speed of the rotor. (*b*) How much mechanical power must be supplied to the frequency converter? (Assume that the converter is perfect.)

CHAPTER XVII

SINGLE-PHASE INDUCTION MOTOR

Any polyphase induction motor may be operated on one phase provided it is first brought up to speed as a polyphase motor or by one of the special methods of starting. Its performance is different from that of the polyphase motor. For example, a three-phase motor when operated on one phase requires approximately $\sqrt{3}$ times the magnetizing current required for three-phase operation while the maximum torque, maximum output, and maximum power factor are much lower.

168. Comparison of the Performance of a Three-phase Induction Motor with the Performance of the Same Motor Operated on One Phase.—A three-phase motor having a Y-connected stator and a Y-connected rotor is selected for the reason that the stator circuit of a commercial single-phase induction motor is almost identical with the circuit obtained by connecting two Y-phases in series.

Consider first two Y-phases 1*A* and 2*A* (see Figs. 135*a* and 135*b*) whose relative motion is such that the mutual inductance between them is proportional to the angle between their axes. i.e., whose mutual inductance is

$$m_a = M \cos \rho t \tag{1}$$

where t is time measured from the instant at which the axes of the two phases coincide, ρ is the relative angular velocity of the two phases about an axis which is common to both. In the illustration $\rho = 2\pi n_2$; where n_2 is the speed of the rotor.

Let a sine-wave current of any frequency f_1 be established in phase 1*A*, and let the phase angle of this current referred to the instant at which the axes of the two phase 1*A* and 2*A* coincide be γ.

The current input may be expressed in the form

$$i_1 = \sqrt{2} I_1 \sin (\omega t - \gamma) \tag{2}$$

where I_1 is its r.m.s. value and $\omega = 2\pi f_1$.

The flux linkages of phase 2A at any instant due to the current i_1 in phase 1A at this instant is from the definition of mutual inductance

$$\lambda_{2a} = m_a i_1 = \sqrt{2} M I_1 \sin(\omega t - \gamma) \cos \rho t$$

$$= \frac{\sqrt{2} M I_1}{2} \sin[(\omega - \rho)t - \gamma] + \frac{\sqrt{2} M I_1}{2} \sin[(\omega + \rho)t - \gamma] \qquad (3)$$

The electromotive force induced in phase 2A by these flux linkages is

$$e_{2a} = \frac{d\lambda_{2a}}{dt} \text{ abvolts}$$

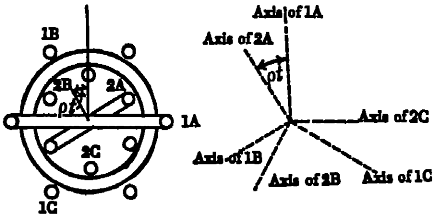
$$= \sqrt{2} \frac{(\omega - \rho) M I_1}{2} \cos[(\omega - \rho)t - \gamma] + \frac{\sqrt{2} (\omega + \rho) M I_1}{2} \cos[(\omega + \rho)t - \gamma]$$

If the rotor is driven at synchronous speed, the electrical time radians $2\pi f_1$ of the stator current in a given time are equal to the electrical space radians $2\pi n_1$ covered by the rotor in the same time. On this basis the slip of the motor may be defined by the relations $s = \frac{\omega - \rho}{\omega}$ and therefore $\rho = (1 - s)\omega$.

This value of ρ substituted in the e.m.f. equation gives for the e.m.f. in phase 2A

$$e_{2a} = \frac{\sqrt{2}}{2} s \omega M I_1 \cos[s \omega t - \gamma] + \frac{\sqrt{2}}{2} (2 - s) \omega M I_1 \cos[(2 - s)\omega t - \gamma] \qquad (4)$$

The electromotive force consists of two components, one having a frequency of sf_1 and the other a frequency of $(2 - s)f_1$. The first component corresponds to the e.m.f. in the rotor of an ordinary polyphase induction motor. The second component corresponds to the e.m.f. in the rotor of an ordinary polyphase induction motor when its rotor is driven backwards by an auxiliary motor. Note also that the r.m.s. value of each component is proportional to its frequency.



(a) (b)

FIG. 135.—Circuits of a Three-phase Induction Motor

Consider now two other secondary phases $2B$ and $2C$, the axis of $2B$ leading the axis of $2A$ by 120 degrees and the axis of $2C$ lagging the axis of $2A$ by 120 degrees, as shown in Fig. 135*b*. The mutual inductances of phase $1A$ with respect to these two phases are then

$$m_b = M\cos(pt+120)$$
$$m_c = M\cos(pt-120)$$

The electromotive forces induced in these two phases by the current i_1 are

$$e_{2b} = \frac{\sqrt{2}s\omega MI_1}{2}\cos[s\omega t-\gamma-120] + \frac{\sqrt{2}(2-s)\omega MI_1}{2}\cos[(2-s)\omega t-\gamma+120] \quad (5)$$

and

$$e_{2c} = \frac{\sqrt{2}s\omega MI_1}{2}\cos[s\omega t-\gamma+120] + \frac{\sqrt{2}(2-s)\omega MI_1}{2}\cos[(2-s)\omega t-\gamma-120] \quad (6)$$

The electromotive forces induced in the three secondary phases therefore each consist of two components, one of frequency sf_1 and one of frequency $(2-s)f_1$. The components having the frequency sf_1 have a phase rotation opposite to those of frequency $(2-s)f_1$.

Let r_2 = the resistance of each of the secondary phases;
L'_2 = self-inductance of each of the secondary phases;
and
M_2 = the mutual inductance between any two of the secondary phases.

Note first that if i_{2a}, i_{2b}, and i_{2c} are any positive currents in the three secondary phases, then the total flux linkages of phase $2A$ due to the current in this phase and the currents in the other two secondary phases are

$$\lambda_{2a} = L'_2 i_{2a} - M_2 i_{2b} - M_2 i_{2c}$$

But in a three-phase circuit $i_{2a}+i_{2b}+i_{2c}=0$. Whence $i_{2b}+i_{2c}=-i_{2a}$, and therefore the total flux linkages of phase $2A$ due to the currents in all three phases of the secondary

$$\lambda_{2a} = (L'_2+M_2)i_{2a} = L_2 i_{2a} \quad (7)$$

Now the total e.m.f. induced in any phase of the secondary by the primary current must be equal to the resistance drop in this phase plus the inductive drop in this phase due to the secondary currents. Whence, for phase 2*A*

$$e_{2a}=r_2i_{2a}+L_2\frac{di_{2a}}{dt} \tag{8}$$

Since e_{2a} consists of two components of different frequencies, the current i_{2a} must likewise consist of two components of the same frequencies, and the component of each frequency must satisfy equation (8) separately. The r.m.s. values of these components of the secondary current will then be respectively [see equation (4)].

$$I'_2=\frac{s\omega MI_1}{2\sqrt{r_2^2+(s\omega)^2L_2^2}} \tag{9}$$

$$I''_2=\frac{(2-s)\omega MI_1}{2\sqrt{r_2^2+(2-s)^2\omega^2L_2^2}} \tag{10}$$

and the angles by which these two components lag the corresponding induced electromotive forces are

$$\theta'_2=\tan^{-1}\left[\frac{s\omega L_2}{r_2}\right] \tag{11}$$

$$\theta''_2=\tan^{-1}\left[\frac{(2-s)\omega L_2}{r_2}\right] \tag{12}$$

Note that

$$\cos\,[s\omega t-\gamma]=\sin\left[s\omega t-\gamma+\frac{\pi}{2}\right]$$

and therefore the expression for the instantaneous value of the first component of the secondary current in phase 2*A* is

$$I'_2\sin\left[s\omega t-\gamma+\frac{\pi}{2}-\theta'_2\right]$$

Similarly the expression for the instantaneous value of the second component of the secondary current in phase 2*A* is

$$I''_2\sin\left[(2-s)\omega t-\gamma+\frac{\pi}{2}-\theta''_2\right]$$

Put

$$\gamma'_2=\frac{\pi}{2}-\gamma-\theta'_2$$

and

$$\gamma''_2=\frac{\pi}{2}-\gamma-\theta''_2$$

Then the instantaneous values of the currents in the three secondary phases are

$$i_{2a}=\sqrt{2}I'_2\sin[s\omega t+\gamma'_2]+\sqrt{2}I''_2\sin[(2-s)\omega t+\gamma''_2] \quad (13)$$

$$i_{2b}=\sqrt{2}I'_2\sin[s\omega t+\gamma'_2-120]$$
$$+\sqrt{2}I''_2\sin[(2-s)\omega t+\gamma''_2+120] \quad (14)$$

$$i_{2c}=\sqrt{2}I'_2\sin[s\omega t+\gamma'_2+120]$$
$$+\sqrt{2}I''_2\sin[(2-s)\omega t+\gamma''_2-120] \quad (15)$$

These secondary currents in turn induce electromotive forces in the primary winding. To obtain an expression for the total electromotive force thus induced in the primary, it is necessary to express first the total flux linkages of the primary due to the secondary currents. The flux linkages of the primary due to the current in phase $2A$ of the secondary are

$$\lambda_{1a}=m_a i_{2a}$$
$$=\sqrt{2}MI'_2\sin[s\omega t+\gamma'_2]\cos pt$$
$$+\sqrt{2}MI''_2\sin[(2-s)\omega t+\gamma''_2]\cos pt$$

Put

$$p=(1-s)\omega$$

and expand. Then

$$\lambda_{1a}=\frac{\sqrt{2}MI'_2}{2}\sin(\omega t+\gamma'_2)+\frac{\sqrt{2}MI''_2}{2}\sin(\omega t+\gamma''_2)$$
$$+\frac{\sqrt{2}MI'_2}{2}\sin[(2s-1)\omega t+\gamma'_2]+\frac{\sqrt{2}MI''_2}{2}\sin[(3-2s)\omega t+\gamma''_2]$$

Similarly, the total flux linkages of the primary due to the current in the phase $2B$ of the secondary are

$$\lambda_{1b}=\frac{\sqrt{2}MI'_2}{2}\sin(\omega t+\gamma'_2)+\frac{\sqrt{2}MI''_2}{2}\sin(\omega t+\gamma''_2)$$
$$+\frac{\sqrt{2}MI'_2}{2}\sin[(2s-1)\omega t+\gamma'_2-240]$$
$$+\frac{\sqrt{2}MI''_2}{2}\sin[(3-2s)\omega t+\gamma''_2+240]$$

Similarly

$$\lambda_{1c}=\frac{\sqrt{2}MI'_2}{2}\sin(\omega t+\gamma'_2)+\frac{\sqrt{2}MI''_2}{2}\sin(\omega t+\gamma''_2)$$
$$+\frac{\sqrt{2}MI'_2}{2}\sin[(2s-1)\omega t+\gamma'_2+240]$$
$$+\frac{\sqrt{2}MI''_2}{2}\sin[(3-2s)\omega t+\gamma''_2-240].$$

The sum λ_{1a}, λ_{1b}, and λ_{1c} gives the total flux linkages of the primary due to the three secondary currents. Note that the sum of the three components which have the frequency $(2s-1)f_1$ is zero since the three components differ in phase by 120 degrees. Also, for the same reason, the sum of the three components which have the frequency $(3-2s)f_1$ is zero. Hence the total flux linkages of phase 1*A* due to the secondary currents are

$$\lambda_1=\frac{\sqrt{2}\,3MI'_2}{2}\sin(\omega t+\gamma'_2)+\frac{\sqrt{2}\,3MI''_2}{2}\sin(\omega t+\gamma''_2) \quad (16)$$

The corresponding counter electromotive force induced in the primary phase 1*A* due to these flux linkages is then

$$e_1=\frac{\sqrt{2}\,3\omega MI'_2}{2}\cos(\omega t+\gamma'_2)+\frac{\sqrt{2}\,3\omega MI''_2}{2}\cos(\omega t+\gamma''_2) \quad (17)$$

Note that the secondary currents, although having two components of frequencies $(2s-1)f_1$ and $(3-2s)f_1$, induce in the primary an electromotive force of the same frequency as the primary current, but that this is true only when the secondary has a polyphase winding. If a single-phase winding were used on the secondary, the electromotive force in the primary would have components of frequency $(2s-1)f_1$ and $(3-2s)f_1$ in addition to the frequency f_1. The currents produced in the primary by these electromotive forces would in turn induce in the secondary currents of other frequencies and a cumulative frequency effect would result.

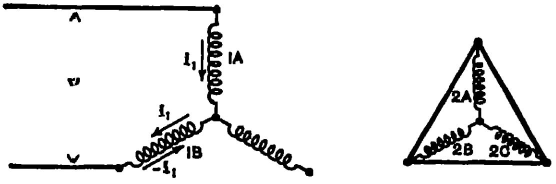
Let r_1 = resistance of a primary phase;
L_1 = total inductance of a primary phase.

Then from equation (17), the voltage which must be impressed on the phase 1*A* in order to produce the primary current i_1 is

$$v_1=r_1i_1+L_1\frac{di_1}{dt}+e_1$$

$$v_1=\sqrt{2}[r_1I_1\sin(\omega t-\gamma)+\omega L_1I_1\cos(\omega t-\gamma)]+e_1 \quad (18)$$

Consider now the case of a three-phase motor having a Y-connected primary, and let this motor be operated as a single-phase motor with two branches of the Y forming the single-phase primary, as shown in Fig. 136. This corresponds to the actual arrangement of windings on a single-phase motor.



Stator **Rotor**

FIG. 136.—Three-phase Induction Motor Operated on One Phase

Let M_1 be the mutual inductance between the two phases of the primary and let L'_1 be the self-inductance of each phase of the primary. If i_1 is the current toward the neutral in phase $1A$, then the current toward the neutral in phase $1B$ is $-i_1$, and therefore the flux linkages of phase $1A$ due to the single-phase primary current in both windings are

$$L'_1 i_1 + M_1 i_1 = (L'_1 + M_1) i_1$$

Hence the equivalent self-inductance of phase $1A$ is (L'_1+M_1) and therefore the equivalent self-inductance of the two primary phases in series is

$$L = 2(L'_1 + M_1) = 2L_1 \qquad (19)$$

The magnetic field due to a current in the two primary phases (Fig. 136) in series is $\sqrt{3}$ times as powerful as that produced by the same current in one phase only. This is evident when we keep in mind that a given current I_1 entering the line terminal of phase $1A$ produces a magnetic field along its axis and that an equal current I_1 entering the line terminal of phase $1B$ produces a magnetic field 120 electrical space degrees displaced from the field produced by $1A$ and of equal strength. It follows that a current I_1 entering the line terminal of phase $1A$ and leaving the line terminal of phase $1B$ produces a magnetic field $\sqrt{3}$ times as powerful as that produced by one phase alone. The axis of the resultant field lags the axis of $1A$ by an angle of 30 electrical space degrees. The two phases in series are equivalent to a single coil whose maximum mutual inductance relative to a rotor phase is $\sqrt{3}M$ and whose magnetic axis lags the magnetic axis of phase 1*A* by 30 degrees.

The expression for the mutual inductance between rotor phase 2*A* and the two primary phases in series now becomes

$$m_a = \sqrt{3}M \cos pt \quad \text{(compare with equation (1))}$$

where pt is now measured from the axis of the coil equivalent to the two primary phases in series.

Let r_1 be the resistance of each of the two phases of the primary. Then the equivalent resistance of the two primary phases in series is

$$r = 2r_1 \tag{20}$$

Equations (1), (4), (5), (6), (9), (10), (17), and (18) then also hold for the connections shown in Fig. 136 provided that to M is assigned the value $\sqrt{3}M$, to L_1 the value $2(L'_1+M_1)=2L_1$ and to r_1 the value $2r_1$, and provided γ is taken as the phase angle of the primary current at the instant when the axis of the secondary phase 2*A* coincides with the axis of the equivalent primary winding i.e., with the line which lags 30 degrees behind the axis of phase 1*A* of the primary.

Substituting $\sqrt{3}M$ in place of M in equation (4), we obtain

$$e_{2a} = \frac{\sqrt{2}}{2} s\omega\sqrt{3}MI_1 \cos(s\omega t - \gamma)$$

$$+\frac{\sqrt{2}}{2}(2-s)\omega\sqrt{3}MI_1 \cos[(2-s)\omega t - \gamma] \tag{21}$$

The two component e.m.f.'s in each phase of the secondary establish currents each of which consists of two components having the r.m.s. values

$$I'_2 = \frac{\omega\sqrt{3}MI_1}{2\sqrt{\left(\frac{r_2}{s}\right)^2 + (\omega L_2)^2}} \tag{22}$$

$$I''_2 = \frac{\omega\sqrt{3}MI_1}{2\sqrt{\left(\frac{r_2}{2-s}\right)^2 + (\omega L_2)^2}} \tag{23}$$

The counter e.m.f. induced in the primary by the secondary currents is [see equation (17)]

$$e_1=\sqrt{2}\frac{3\omega\sqrt{3}MI'_2}{2}\cos(\omega t+\gamma'_2)$$

$$+\sqrt{2}\frac{3\omega\sqrt{3}MI''_2}{2}\cos(\omega t+\gamma''_2) \tag{24}$$

The required primary voltage is

$$v_1=\sqrt{2}[2r_1I_1\sin(\omega t-\gamma)+2\omega L_1I_1\cos(\omega t-\gamma)]+e_1 \tag{25}$$

Consider now two identical single-phase transformers, each having a primary resistance r_1, secondary resistance $\frac{1}{3}r_2$, total primary inductance L_1, total secondary self-inductance $\frac{1}{3}L_2$, and mutual inductance $\frac{\sqrt{3}M}{2}$, and with a turn ratio equal to $\sqrt{3}$.

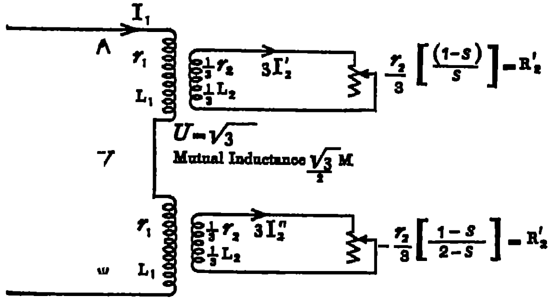


FIG. 137.—Equivalent Circuit of a Three-phase Induction Motor Operated on One Phase

Let the primaries of these two transformers be connected in series (Fig. 137) and let the secondary of the first one be connected to a non-inductive load of resistance $\frac{r_2}{3}\left(\frac{1-s}{s}\right)$ making the total resistance of its secondary $\frac{1}{3}\frac{r_2}{s}$, and let the secondary of the other transformer be connected to a non-inductive load of resistance $-\frac{r_2}{3}\left(\frac{1-s}{2-s}\right)$ making the total resistance of the secondary $\frac{r_2}{3}\left(\frac{1}{2-s}\right)$

By applying to Fig. 137 the same method of analysis as was used for the single-phase motor (noting that the mutual inductance for the transformer is constant, which greatly simplifies the analysis), it may be shown that when a current

$$i_1 = \sqrt{2} I_1 \sin(\omega t - \gamma)$$

is established in the two primary windings that the current in the upper secondary will have an r.m.s. value equal to three times the r.m.s. value of the low-frequency secondary current in the single-phase motor.

The current in the lower secondary will have an r.m.s. value equal to three times the r.m.s. value of the high-frequency secondary current in the single-phase motor.

The back induced e.m.f. in the upper primary will be the same as the first component of equation (24).

The back induced e.m.f. in the lower primary will be the same as the second component of equation (24).

The power input to the upper secondary will be equal to that part of the total power input to all three secondary phases of the single-phase motor due to the low-frequency secondary currents.

The power input to the lower secondary will be equal to that part of the total power input to all three secondary phases of the single-phase motor due to the high-frequency secondary currents.

The power input to the external secondary resistance R'_2 which is equal to the power input to the upper secondary less the copper loss in this secondary, is equal to the mechanical output of the rotor due to the low-frequency currents.

The power absorbed in R''_2 is a negative quantity, and therefore represents an electric power output. It is equal to the reduction in the mechanical output due to the presence of the secondary currents of high frequency.

The magnetic reactions in the transformer are identical with those in the motor and therefore, the primary power factor, primary voltage, and primary current are identically the same.

Figure 137 may therefore be looked upon as a circuit electrically equivalent to the single-phase motor, the power input to the resistance R'_2 and R''_2 being equal to the mechanical output of the motor.

Note that any transformer which has a primary winding of total self-inductance L_1, secondary winding of total self-inductance L_2, mutual inductance M, turn ratio $U\left(\frac{N_H}{N_L}\text{ is assumed equal to }\frac{N_1}{N_2}\right)$, and primary and secondary resistances r_1 and r_2, may be represented by a single circuit diagram as shown in Fig. 138, where the constants x_1, x_{21}, r_1 and r_{21} have the values indicated on the diagram. The core-loss conductance and the magnetizing susceptance are represented by the symbols g_1 and b_1 respectively.

The constants of the exactly equivalent circuit (Fig. 139) corresponding to the transformer shown in Fig. 137 are obtained by substituting for the turn ratio the $\sqrt{3}$, for the mutual inductance $\frac{\sqrt{3}}{2}M$, for the inductance L_2 the value $\frac{L_2}{3}$, and for r_2 the value $\frac{r_2}{3}$.

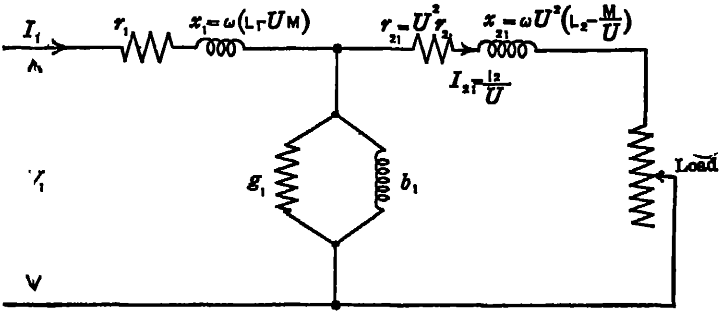


FIG. 138.—Equivalent Circuit for a Transformer

The values of r_1, r_{21}, x_1, x_{21}, g_1, and b_1 in Fig. 139 are identical with the values per phase of the corresponding quantities for a

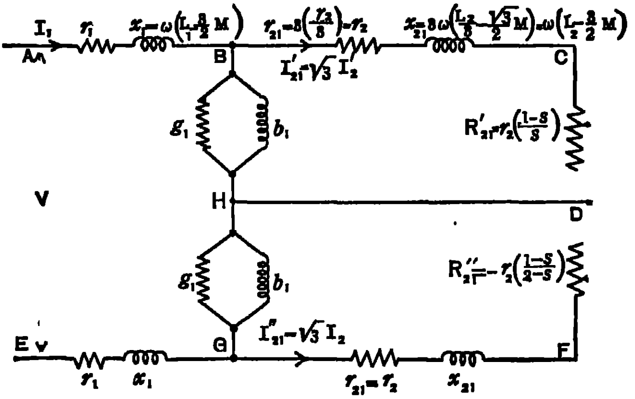

I_1
r_1
$x_1 = \omega(L_1 - \frac{3}{2}M)$
$r_{21} = 3(\frac{r_2}{3}) = r_2$
$x_{21} = 3\omega(\frac{L_2}{3} - \frac{\sqrt{3}}{2}M) = \omega(L_2 - \frac{3}{2}M)$
A
B
C
$I'_{21} = \sqrt{3}\, I_2$
g_1
b_1
$R'_{21} = r_2(\frac{1-s}{s})$
V
H
D
$R''_{21} = -r_2(\frac{1-s}{2-s})$
$I''_{21} = -\sqrt{3}\, I_2$
E
G
F
r_1
x_1
$r_{21} = r_2$
x_{21}

FIG. 139.—Equivalent Circuit of a Three-phase Induction Motor Operated on One Phase

Y-connected motor operated as a three-phase motor. That is, r_1 and x_1 are the resistance and leakage reactance per phase of the primary, r_{21} and x_{21} are the resistance and leakage reactance per phase of the secondary reduced to primary turns, and g_1, and b_1, are the core-loss conductance and magnetizing susceptance per phase referred to the primary.

Referring to Fig. 139, note that the resistance R''_{21} lies between $-\frac{1}{2}r_2$ and 0 for any value of the slip between zero and unity, and, therefore, the resistance of the branch *DFG* of the network lies between $\frac{1}{2}r_2$ and r_2 for all values of the slip between zero and unity. The impedance of the branch *DFG* is therefore small compared with the impedance of the branch *HG*, and hence the complete removal of the branch *HG* produces but a slight effect in the values of the currents in the other branches of the network. The potential difference between *D* and *G* will also be relatively small compared with the impressed voltage. Hence, with the branch *HG* removed, *H* may, with but small error, be considered as connected directly to *G* instead of to *D*.

The resistance R''_{21} then becomes directly in series with R'_{21} and their joint resistance is

$$R_{21}=r_2\left[\frac{1-s}{s}-\frac{1-s}{2-s}\right]=r_2\left[\frac{2-4s+2s^2}{2s-s^2}\right]$$

For small values of slip s^2 is negligible and

$$R_{21}=2r_2\left[\frac{1-2s}{2s}\right]=2r_2\left[\frac{1-s'}{s'}\right],$$

where $s'=2s$.

On these assumptions, Fig. 139 reduces to Fig. 140, which for small values of the slip, is a fair representation of the single-phase motor shown schematically in Fig. 136.

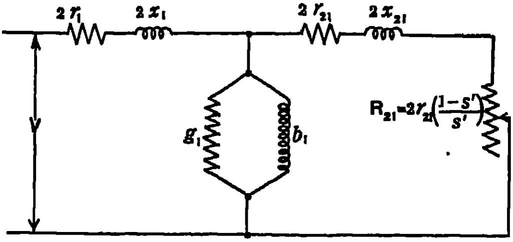


FIG. 140.—Equivalent Circuit of a Single-phase Induction Motor

From Fig. 140 and the corresponding circuit (Fig. 117, Chapter XIII) for a three-phase motor, a comparison may be made of the performance of the given motor when operating single-phase and when operating three-phase.

1. For the same line voltage in both cases the voltage to neutral for three-phase operations is $\frac{V}{\sqrt{3}}$ and the magnetizing current is $b_1\frac{V}{\sqrt{3}}$. The magnetizing current for single-phase operation is b_1V. Hence, when operating single-phase, the magnetizing current is $\sqrt{3}$ times the magnetizing current per phase for three-phase operation.

2. The total magnetizing volt-amperes (all three phases) for three-phase operation are $3\left(b_1\frac{V}{\sqrt{3}}\right)\frac{V}{\sqrt{3}}=b_1V^2$, and the magnetizing volt-amperes for single-phase operation are b_1V^2. The magnetizing volt-amperes are therefore the same in both cases.

3. The total core loss for three-phase operation is $3g_1\left(\frac{V}{\sqrt{3}}\right)^2$ $=g_1V^2$ which is the same as the core loss for a single-phase operation.

4. From equation (24), Chapter XIII the maximum torque for three-phase operation is

$$T_{\text{max}}=\frac{550}{746(2\pi n_1)}(3)\left(\frac{V}{\sqrt{3}}\right)^2\frac{1}{2\{r_1+\sqrt{r_1^2+(x_1+x_{21})^2}\}}$$

Applying the same relation to Fig. 140 the maximum torque for single-phase operation is

$$T'_{\text{max.}}=\frac{550}{746(2\pi n_1)}(1)(V)^2\frac{1}{4\{r_1+\sqrt{r_1^2+(x_1+x_{21})^2}\}}$$

Hence for single-phase operation the maximum torque is one-half the maximum torque for three-phase operation.

5. Applying equation (23), Chapter XIII, to both cases, the value of s' in Fig. 140 at which maximum torque occurs for single-phase operation is equal to the actual slip at which maximum torque occurs for three-phase operation. Hence the actual slip at which maximum torque occurs for single-phase operation is one-half the slip at which maximum torque occurs for three-phase operation. *Note*: Due to the approximation involved in the equivalent circuit shown in Fig. 140 the relations expressed in

(4) and (5) are only roughly approximate. Actually, the maximum torque and the maximum power output for single-phase operation are about 40 per cent, instead of 50 per cent, of their values for three-phase operation.

6. In order to keep the same ratio between the maximum torque and the rated torque, for the same line voltage, a three-phase motor when intended for single-phase operation must therefore be given a rating of about 40 per cent of its three-phase rating. The current taken by the motor at this reduced rating will be slightly less than that corresponding to its three-phase rating, and consequently the total heating will be less. For this reason a higher voltage (usually about 30 per cent higher than the three-phase line voltage) may therefore be used without exceeding the safe temperature limits. Since the maximum torque is proportional to the square of the impressed voltage, the single-phase rating of the motor is then increased 70 per cent above its single-phase rating at three-phase line voltage, resulting in a single-phase rating of $40 \times 1.70 = 68$ per cent of its three-phase rating. In short, the temperature rise of a three-phase motor when operated single-phase at approximately 70 per cent of its three-phase rating, with the primary impressed voltage 30 per cent in excess of the three-phase line voltage, will be the same as when operated as a three-phase motor, and the ratio of the maximum torque to rated torque will be the same both single-phase and three-phase. The power factor and efficiency, however, will be less for single-phase than for three-phase operation.

169. Methods of Starting Single-phase Motors.—

1. The motor may be started by repulsion motor action (see next Article). When the proper speed is reached the commutator is automatically short-circuited by a short-circuiting ring and the brushes are automatically lifted off the commutator.

2. The proper starting torque may be obtained by using shading coils in addition to the main stator field coils (see Fig. 141). The field flux in the unshaded portion of the pole leads the field flux in the shaded portion of the pole. A shifting magnetic field is obtained which induces currents in the rotor and produces a starting torque.

3. Starting torque may also be obtained by using a phase-splitting device (see Fig. 142). By this scheme a revolving field is produced and starting torque is obtained. After the rotor has

gained momentum the phase-splitting device may be disconnected and the two stator windings may be connected in parallel.

170. Repulsion Motor.—A repulsion motor is a form of single-phase motor in which the rotor is similar to the armature of an ordinary d-c. motor. The brushes bearing on the commutator are short-circuited.

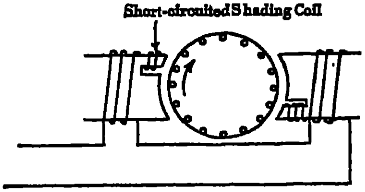


FIG. 141.—Shaded Pole Single-phase Motor

Before giving a detailed explanation of the operation of a repulsion motor it is essential to know that the armature of an ordinary d-c. motor is magnetically equivalent to a coil having its axis parallel to the axis of the brushes. When the brushes are shifted the armature field is shifted a like amount. To illustrate this point consider the two-pole ring wound armature in Fig. 143. A current entering brush *A* and leaving brush *B* produces a field down while a current entering brush *A′* and leaving brush *B′* produces a field to the left. A shift in the brushes from *A* to *A′* and *B* to *B′* results in a like shift of the armature field. The complicated armature winding of a d-c. motor may therefore be represented by a magnetically equivalent winding (Fig. 144).

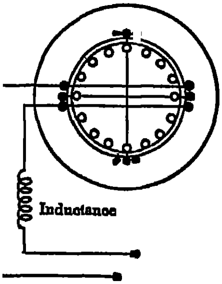


FIG. 142.—Phase Splitting Device

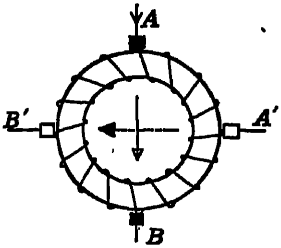


FIG. 143.—Location of Armature Reaction Field Relative to Brush Position of a Single-phase Motor

171. Operation of the Repulsion Motor.—The operation of a repulsion motor may be studied by reference to Figs. 145*a*, 145*b*, 145*c*, and 145*d*. These figures differ only in the position of the short-circuited brushes. When the axis of the brushes is parallel to the axis of the stator field a large current is induced in the rotor by transformer action. This current produces a magnetic reaction directly opposed to the field produced by the stator current alone and no torque is possible. If the axis of the brushes is perpendicular to the axis of the stator field and the rotor is stationary no currents are induced in the rotor and no torque results. When the brushes are shifted to the position indicated in Fig. 145*c*, the rotor field reacts on the stator field (the rotor poles repel the stator poles) and rotation takes place in a counter-clockwise direction. To reverse the direction of rotation it is necessary to shift the brushes to the position indicated in Fig. 145*d*.

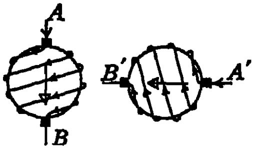


FIG. 144.—Armature Winding Equivalent to the Armature Winding of a Repulsion Motor

172. Voltage-Current Vector Diagram for a Repulsion Motor.— In Fig. 145*c* a current in the primary winding *A B* produces an alternating flux, part of which may be considered as a mutual flux linking both windings *AB* and *CD*. On account of this mutual flux e.m.f.'s are induced in both windings by transformer action. These windings are similar to those on an ordinary power transformer; i.e., the e.m.f.'s due to transformer action lag the mutual flux by 90 degrees (see Fig. 146). In addition to the transformer e.m.f. in the circuit *CD* there is another e.m.f. due to the fact that the rotor winding cuts the magnetic field produced by the primary current. The cutting e.m.f. or rotational e.m.f. as it is often called is positive when the flux is positive and has a maximum value when the flux has a maximum value. For these reasons the cutting e.m.f. may be represented by a vector in phase with the flux. The resultant e.m.f. *CD* sets up a current I_{CD} which lags the e.m.f. by an angle which is determined by the resistance and the leakage reactance of the rotor winding. The rotor current referred to the primary winding when combined with the no-load current gives the total primary current. The impressed voltage on the primary winding is equal to the vector sum of the counter

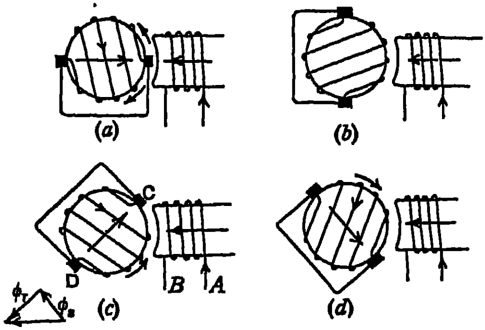


FIG. 145.—Diagrams Showing that the Torque Developed in a Repulsion Motor Depends on the Location of the Brushes

voltage due to the mutual flux, the resistance drop, and the primary winding leakage reactance drop.

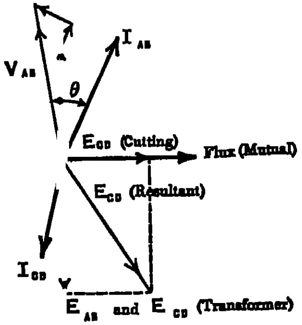


FIG. 146.—Vector Diagram for the Repulsion Motor in Fig. 145 (c)

173. Power Factor Compensation in Repulsion Motors.—The stator field of the motor in Fig. 145*c* may be replaced by two coils having their axis 90 electrical space degrees apart (see Fig. 147). Coil No. 1 produces the transformer field ϕ_T which links the armature circuit *CD* and produces a current in it by transformer action. Coil No. 2 produces a field which reacts on the currents in *CD* and produces torque. The field produced by coil No. 2 is responsible for the speed of rotation and for this reason it is often called the speed field ϕ_s.

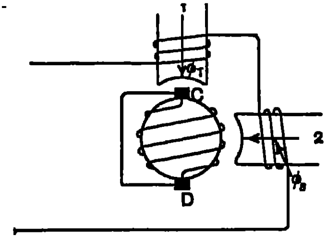


FIG. 147.—Transformer Field and Speed Field of a Repulsion Motor

Coil No. 2 may be eliminated from the stator winding by adding another set of brushes *EF* located 90 electrical space degrees from the brushes *CD* (Fig. 148) and connected in series with coil No. 1. In other words the armature itself may be made to produce the speed field ϕ_s. The motor in Fig. 148 has the disadvantage of having more brushes than an ordinary repulsion motor, but it has the decided advantage in that it operates at a better power factor. The circuit *EF* has two e.m.f.'s induced in it, one due to the transformer action of the flux ϕ_s and the other due to cutting of the flux ϕ_T. These e.m.f.'s have components which respectively lag and lead the current through *EF* (see Figs. 149 and 150). It follows that the reactance of the circuit *EF* may be made positive or negative, depending on the relative magnitudes of these

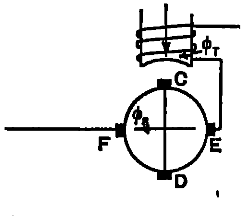


FIG. 148.—Diagram Showing How the Speed Field ϕ_s of a Repulsion Motor May be Produced by the Armature Winding

e.m.f.'s. Since the reactance of coil No. 1 is positive it may be compensated by a negative reactance in the circuit *EF* and the motor may be operated at any desired power factor.

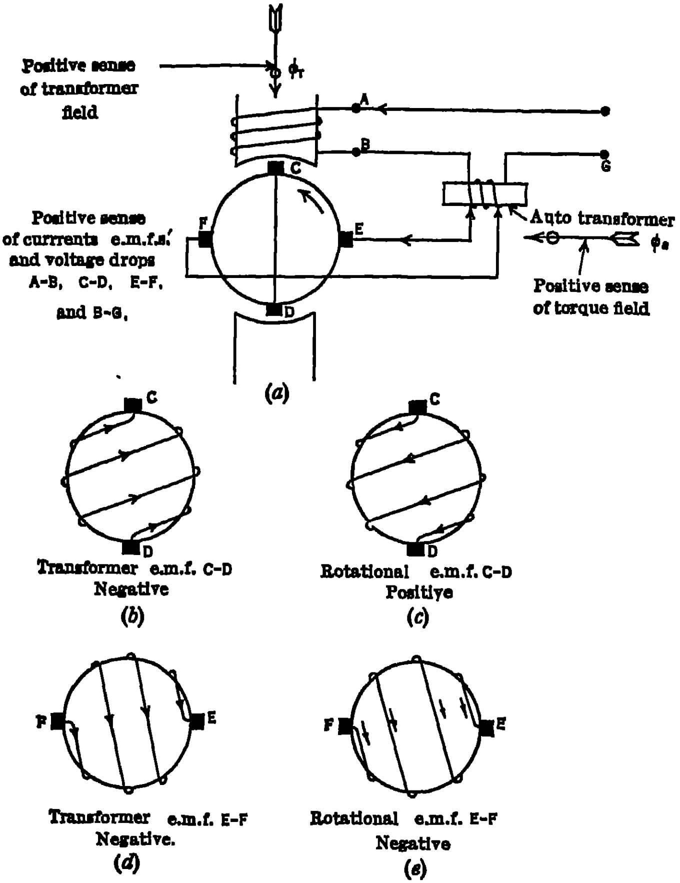

Positive sense of transformer field
ϕ_T
A
B
C
G
Positive sense of currents e.m.f.s. and voltage drops A-B, C-D, E-F, and B-G,
F
E
D
Auto transformer
ϕ_s
Positive sense of torque field
(a)
C
D
Transformer e.m.f. C-D Negative
(b)
C
D
Rotational e.m.f. C-D Positive
(c)
F
E
Transformer e.m.f. E-F Negative.
(d)
F
E
Rotational e.m.f. E-F Negative
(e)

FIG. 149.—Directions of Induced E.M.F.'s in a Compensated Repulsion Induction Motor

174. Winter-Eichberg-Latour Compensated Repulsion Induction Motor.—One of the fundamental methods for obtaining good power factor on single-phase induction motors is illustrated by Figs. 149 and 150.

The motor is so connected that counter-clockwise rotation is obtained. Two e.m.f.'s are set up in each of the circuits *CD* and *EF*. The directions of these e.m.f.'s for the assumed positive directions of the fields ϕ_s and ϕ_T are indicated in Figs. 149*b*–149*e*. The voltage drop *BG* is equal to some constant multiplied by the resultant e.m.f. *EF*. By properly adjusting the rotational e.m.f. *EF*, it is possible to obtain a high power factor on the primary circuit.

175. Shunt and Series Motors Operated by Single-phase Alternating-current.—Ordinary d-c. shunt and series motors cannot be operated by single-phase alternating-current for the following reasons:

1. Per ampere input the torque developed by alternating-current is much smaller than that obtained per ampere of direct-current.
2. The power factor is very low.
3. The eddy-current losses are very large.
4. The commutation is very poor.

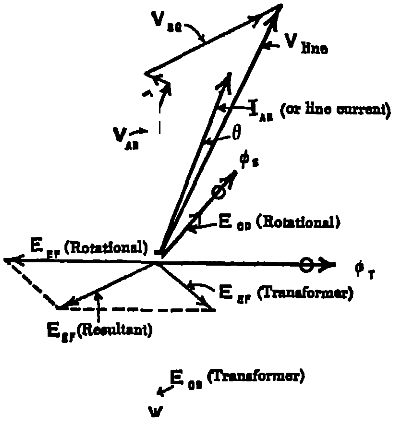


FIG. 150.—Vector Diagram for the Compensated Repulsion–Induction Motor Shown in Fig. 149

The shunt motor operated by alternating-current.—The inductance of the field winding of an ordinary d-c. shunt motor is so high that with an alternating-voltage applied the field current is extremely small, the field flux is small and the torque which depends on the product of the field flux and the armature current is necessarily very small.

Both the armature and the field circuits are highly inductive and for this reason the power factor must be low.

A solid field yoke subjected to an alternating-flux acts like a short-circuited secondary winding on a transformer with the result that the iron losses in a solid yoke are excessive.

A coil undergoing commutation has the following component e.m.f.'s induced in it:

1. The e.m.f. of self-induction.
2. The e.m.f. due to cutting of the main field.
3. The e.m.f. due to transformer action. The axis of a coil undergoing commutation is parallel to the axis of the main field

and acts like a short-circuited secondary winding on a transformer (see Fig. 151).

The e.m.f. due to transformer action is responsible for the commutating difficulties in single-phase commutating machines.

The d-c. series motor operated by alternating-current.—The d-c. series motor operated by alternating-current has the same faults as the shunt motor, but the torque developed per ampere input is considerably greater.

The average torque developed depends on the maximum value of the field flux, the maximum value of the armature current and the time phase relations between them. In a series motor the field current and the armature current are necessarily in phase and equal. On account of the field yoke losses the current and the field flux cannot be in phase and for this reason the torque per ampere of input is small compared to that obtained when the motor is operated by direct-current.

The a-c. series motor.—A series motor if properly designed may be operated by alternating-current. A good a-c. series motor has the following design features:

1. The field yoke is laminated.
2. The field reluctance is small.
3. The number of field turns are small.
4. The flux density in the magnetic circuit is low.
5. A compensating winding is provided.
6. The number of turns on the armature is greater than those on an ordinary d-c. motor of the same armature diameter and starting torque.
7. The number of armature turns per commutator segment i smaller than in a d-c. motor.
8. The number of commutator segments is large.
9. Special provisions are made for obtaining good commu tation.

In order to have the field flux nearly in phase with the field or armature current it is necessary that the losses in the field yok are small. This requires that the magnetic circuit be completel laminated.

To obtain a small inductance for the field winding it is necessar that the winding has few turns and that the flux linking it is smal The minimum inductance for a given field flux is obtained whe the reluctance of the magnetic circuit has the smallest possibl value. For this reason an a-c. series motor has a more compact magnetic circuit and is operated at a lower value of field flux than a d-c. motor of equal speed and power.

The torque of a motor depends on the product of the field flux and the armature flux. If the field is designed for a relatively small flux it is necessary to construct the armature magnetically more powerful; i.e., for a given torque more turns are required on the armature of a single-phase motor than on the armature of a d-c. motor of equal speed and power.

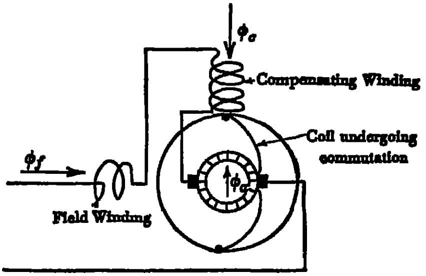


FIG. 151.—Compensated A-c. Series Motor

To obtain a high power factor for the armature circuit a compensating winding is required (see Fig. 151). The compensating winding is magnetically opposed to the armature winding with the result that the total reactance of the armature and compensating winding combined is small.

The commutation difficulties are overcome by connecting the commutator segments to the armature coils through resistance leads (see Fig. 152). The resistance leads serve the double purpose of reducing the short-circuit current in a coil undergoing commutation and at the same time decrease the rI^2 losses due to commutation. The total load current flows through several resistance leads in parallel; but the short-circuit current flows through two resistance leads in series. The added rI^2 losses in the resistance leads due to the load current is more than offset by the reduction in the short-circuit rI^2 losses.

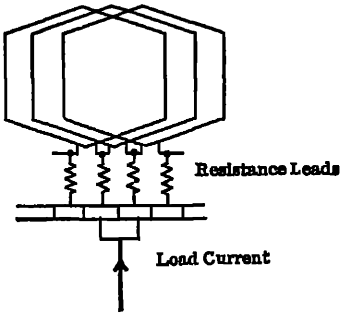


FIG. 152.—Resistance Leads for Improving Commutation

176. Voltage Drop-Current Vector Diagram for an A-C. Series Motor (Fig. 153).—The total impressed voltage on an a-c. series motor may be expressed by the relation:

$$\underset{.}{V}=(R+jX)\underset{.}{I}+\underset{.}{E}$$

where R = resistance of field winding + resistance of compensating winding + resistance of armature winding $= r_f + r_c + r_a$;

X = total field reactance + compensating winding leakage reactance + armature leakage reactance $= x_f + x_c + x_a$

and

E = armature rotational voltage.

The drop due to the armature rotational voltage is practically in phase with the armature current for the reasons that the field flux is nearly in phase with the armature current and that the flux cut by the armature conductors is large when the field flux is large.

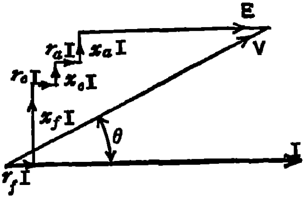


FIG. 153.—Vector Diagram for an A-c. Series Motor

An examination of Fig. 153 shows that the power factor at which an a-c. series motor operates depends on the constants of the motor and on the speed of the motor. At starting, the rotational e.m.f. is zero and the power factor is low. As the speed increases the power factor gets better and better, but on account of the field reactance it must always be less than unity.

PROBLEMS

1. A 4-pole, 60-cycle, single-phase induction motor is operated at a speed of 1750 r.p.m. Calculate the frequencies of the voltages induced in its rotor winding.

2. The induction motor specified in Problem 7—Chapter XIII is operated with one of the line wires disconnected. The line voltage is 110 volts and the frequency is 60 cycles. (*a*) Calculate the maximum theoretical torque (rotor short-circuited) and the corresponding speed of the motor. (*b*) Calculate the power factor and the line current for the conditions in part (*a*).

CHAPTER XVIII

UNBALANCED POLYPHASE LOADS—PHASE BALANCER

Single-phase loads when supplied by a polyphase generator have a tendency to unbalance the voltage, and the current output of the generator. The magnitude of the unbalance depends on the nature of the single-phase load and on the constants of the generator supplying the load. Unbalanced phase voltages and phase currents are undesirable for they prevent the utilization of the rated capacity of the generator.

The phase voltages and currents of a polyphase generator may be balanced by connecting in parallel with it a machine known as a phase balancer. The phase balancer receives power from the phases of the generator which are under loaded and delivers power to the phases which are over loaded. The power transfer takes place in a machine very similar to an ordinary synchronous motor. In fact any polyphase motor acts more or less like a phase balancer.

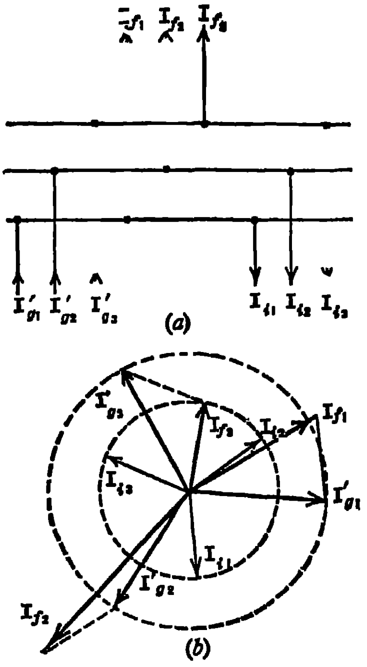


FIG. 154.—Diagram Showing that an Unbalanced Three-phase Load Combined with a Balanced Three-phase Load of Reverse Phase Rotation May Result in a Balanced Load

It is well known that an unbalanced load connected in parallel with a balanced load in which the phase rotation of the currents is different may result in a balanced load. Thus in Fig. 154 the addition of the unbalanced feeder currents I_{f1}, I_{f2}, and I_{f3} to the balanced currents I_{t1}, I_{t2}, and I_{t3} of opposite phase rotation results in balanced generator currents I'_{g1}, I'_{g2}, and I'_{g3}.

The complete phase balancer consists of two synchronous machines with their rotors fastened to a common shaft and with their stator windings connected in series. One of the machines is designed for full line voltage and is large compared to the second machine which is designed for a much lower voltage. The two machines are connected unsymmetrically; i.e., they produce torques in the opposite directions. In actual operation the large machine acts as a motor-generator and the small one as a synchronous generator.

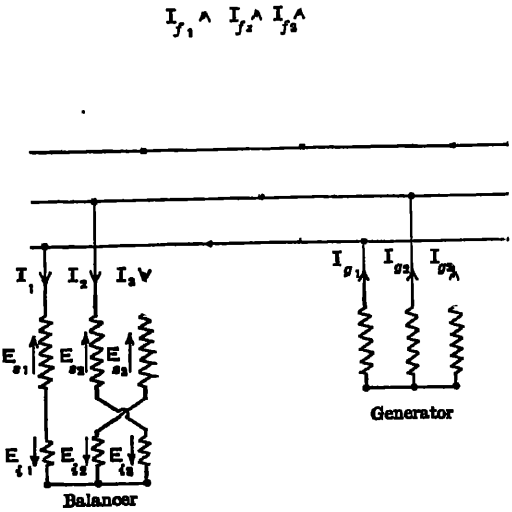


FIG. 155.—Diagram of Connections of a Phase Balancer

By proper field excitation of the two machines of the balancer it is possible to make the balancer take approximately balanced currents from the bus-bars and of phase rotation opposite to that of the bus-bar voltages (bus-bar to neutral) or the unbalanced feeder currents. The windings of the balancer are connected as shown by Fig. 155

The currents in the three phases of the balancer may be expressed by the following equations:

$$\underset{\cdot}{I}_1=\frac{\underset{\cdot}{V}_1-\underset{\cdot}{E}_{s1}}{\underset{\cdot}{Z}}+\frac{\underset{\cdot}{E}_{t1}}{\underset{\cdot}{Z}}=\underset{\cdot}{I}_{s1}+\underset{\cdot}{I}_{t1} \tag{1}$$

$$\underset{\cdot}{I}_2=\frac{\underset{\cdot}{V}_2-\underset{\cdot}{E}_{s2}}{\underset{\cdot}{Z}}+\frac{\underset{\cdot}{E}_{t3}}{\underset{\cdot}{Z}}=\underset{\cdot}{I}_{s2}+\underset{\cdot}{I}_{t2} \tag{2}$$

$$\underset{\cdot}{I}_3=\frac{\underset{\cdot}{V}_3-\underset{\cdot}{E}_{s3}}{\underset{\cdot}{Z}}+\frac{\underset{\cdot}{E}_{t2}}{\underset{\cdot}{Z}}=\underset{\cdot}{I}_{s3}+\underset{\cdot}{I}_{t3} \tag{3}$$

where the symbol V represents the impressed voltage (bus-bar to neutral), E_s represents the excitation e.m.f. of the large machine and E_t the excitation e.m f. of the small machine.

The equations show that the current input to a phase of the balancer consists of two components, one similar to the current input to a synchronous motor and one similar to the current in a short-circuited generator. The equations also show that the currents I_{s1}, I_{s2}, and I_{s3} have the same phase rotation as the bus-bar voltages while the currents I_{t1}, I_{t2}, and I_{t3} have the phase rotation of E_{t1}, E_{t3}, and E_{t2} which is opposite to that of V_1, V_2, and V_3. A vector diagram showing the phase relations between these quantities is given in Fig. 156.

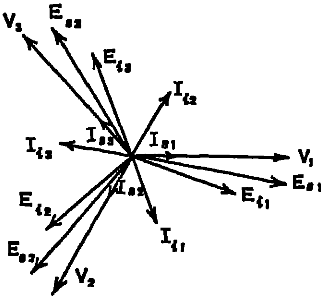


FIG. 156.—Vector Diagram of a Phase Balancer

By proper excitation of the large machine it is possible to make the currents I_{s1}, I_{s2}, and I_{s3} very small and then

$$I_1 = I_{t1}$$

$$I_2 = I_{t2}$$

$$I_3 = I_{t3} \quad \text{(approximately)}$$

The inverse current required to balance an unbalanced three-phase load may be calculated by the following method:

Given: I_{f1} and I_{f2}, the currents in the two feeders of the unbalanced three-phase load. (The current in the third feeder is readily calculated.)

θ_{f1} = angle by which I_{f1} lags or leads the voltage V_1 (the voltage between bus-bar No. 1 and the neutral);

θ_{f2} = the angle between I_{f2} and V_2.

From Fig. 155 it follows that

$$\underset{\cdot}{I}_{g1} = \underset{\cdot}{I}_{f1} + \underset{\cdot}{I}_{t1} + \underset{\cdot}{I}_{s1} \tag{4}$$

$$\underset{\cdot}{I}_{g2} = \underset{\cdot}{I}_{f2} + \underset{\cdot}{I}_{t2} + \underset{\cdot}{I}_{s2} \tag{5}$$

and

$$\underset{\cdot}{I}_{g3} = \underset{\cdot}{I}_{f3} + \underset{\cdot}{I}_{t3} + \underset{\cdot}{I}_{s3} \tag{6}$$

Let

$$(\underset{\cdot}{I}_{g1}-\underset{\cdot}{I}_{s1})=\underset{\cdot}{I}'_{g1}=\underset{\cdot}{I}_{f1}+\underset{\cdot}{I}_{t1} \tag{7}$$

$$(\underset{\cdot}{I}_{g2}-\underset{\cdot}{I}_{s2})=\underset{\cdot}{I}'_{g2}=\underset{\cdot}{I}_{f2}+\underset{\cdot}{I}_{t2} \tag{8}$$

and let

$$\rho=-\tfrac{1}{2}+j\frac{\sqrt{3}}{2} \tag{9}$$

(The product of ρ and the symbolic expression for a vector results in the symbolic expression for an equal vector 120 degrees ahead of the original vector).

From Fig. 154, it is apparent that

$$\underset{\cdot}{I}'_{g2}=\rho^2\underset{\cdot}{I}'_{g1} \tag{10}$$

and

$$\underset{\cdot}{I}_{t2}=\rho I_{t1} \tag{11}$$

Multiplying equation (7) by ρ^2, we obtain

$$\rho^2\underset{\cdot}{I}'_{g1}=\rho^2\underset{\cdot}{I}_{f1}+\rho^2\underset{\cdot}{I}_{t1} \tag{12}$$

and substituting the proper values in (8), we obtain

$$\rho^2 I'_{g1}=\underset{\cdot}{I}_{f2}+\rho\underset{\cdot}{I}_{t1} \tag{13}$$

From (12) and (13), we obtain

$$(\rho^2-\rho)\underset{\cdot}{I}_{t1}=-\rho^2\underset{\cdot}{I}_{f1}+\underset{\cdot}{I}_{f2} \tag{14}$$

and

$$\underset{\cdot}{I}_{t1}=\frac{j\underset{\cdot}{I}_{f2}-j\rho^2 I_{f1}}{\sqrt{3}} \tag{15}$$

Equation (15) enables one to calculate the value of the inverse current required to balance the generator output.

The vector diagram, Fig. 157, shows how the value of I_{t1} and I'_{g1} may be obtained graphically.

The actual generator current is obtained by adding I'_{g1} and $\underset{\cdot}{I}_{s1}$; i.e.,

$$\underset{\cdot}{I}_{g1}=\underset{\cdot}{I}'_{g1}+\underset{\cdot}{I}_{s1} \tag{16}$$

The value of the inverse current is controlled by the excitation of the small machine. From equation (1) it is apparent that the excitation required on the small unit may be calculated by the relation

$$\underset{\cdot}{E}_{t1}=\underset{\cdot}{Z}\underset{\cdot}{I}_{t1} \tag{17}$$

where Z is the phase impedance of the windings of both machines in series. The relation shows that the purpose of the small unit is purely to overcome the impedance drop due to the inverse current.

The phase position of the current I_{i1} with respect to the bus-bar voltages is changed by changing the phase position of the excitation voltage E_{i1}. To accomplish this the rotor of the small machine is provided with two field windings 90 electrical space degrees apart. By adjusting the relative excitations in the two windings it is possible to control the phase of the excitation e.m.f. of the generator element of the balancer.

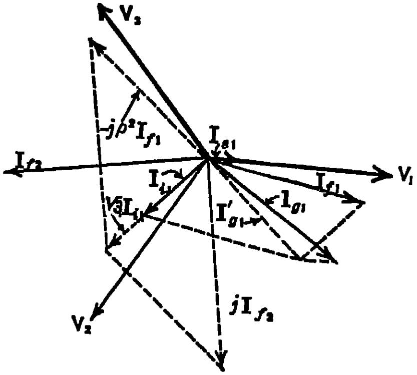


FIG. 157.—Graphical Method for Obtaining the Inverse Current Required to Balance the Output of a Generator

The power input to the balancer must be sufficient to supply the total losses. In a three-phase machine the total input is $3VI_s \cos\theta_s$ where V is the phase voltage, I_s the component of the current which has the same phase rotation as the bus-bar voltages, and $\cos\theta_s$ the phase power factor (θ_s is the angle between V and I_s of a given phase).

The power input due to the inverse components of the current is

$$V_1 I_{i1} \cos\theta_i + V_2 I_{i2} \cos(120+\theta_i) + V_3 I_{i3} \cos(120-\theta_i) = 0$$

The copper loss due to the inverse currents is $3rI_i^2$ and is supplied by motor action in the large machine.

PROBLEMS

1. Two balanced three-phase loads A and B are connected in parallel to the terminals of a three-phase generator. Load A requires 100 amperes (line current) and the phase rotation of the currents is the same as that of the generator voltages. Load B also requires 100 amperes (line current) but in this case the phase rotation of the currents is opposite that of the generator voltages. The phase rotation of the generator voltages (line to neutral) is $V_{10} - V_{20} - V_{30}$. The line current in phase 1–0 of the generator is 200

amperes. (*a*) Calculate the currents in the other phases of the generator. (*b*) Construct a vector diagram showing the voltage and current relations.

2. A certain phase-balancer is used for balancing the output of a three-phase generator. The generator and the balancer are connected to bus-bars numbered 1, 2, and 3. Three feeders A, B, and C, supply the total load and are connected to bus-bars, 1, 2, and 3, respectively. The current in feeder A referred to the bus-bar voltage from bus-bar 1 to neutral is $100 + j115$ amperes; the current in feeder B referred to the same voltage is $-210 + j70$ amperes. (*a*) Calculate: (1) the current in feeder C; and (2) the value of the inverse-phase rotational current required to balance the generator output. (*b*) Calculate the relative capacities of the motor and the generator elements of the balancer. Assume the total power loss in the balancer 10 kw. and the Y-phase voltage (bus-bar to neutral) 1320 volts.

CHAPTER XIX

MEASURING INSTRUMENTS

177. Selection of Meters for a Given Test.—Since the torque developed in the indicating element of practically all a-c. meters is a function of the square of the current in its coils and a function of the location of the indicating element with respect to the stationary element, it follows that the scale of these meters is non-uniform. The points of division are usually crowded more at the ends of the scale than at the middle. The probability that a given instrument is accurate when its indication is near the lower end of the scale is not very great. For this reason the meters for a given test should be so chosen that deflections well up on the scale are obtained. One very good way for overcoming part of this difficulty is to use 5-ampere ammeters and wattmeters in connection with a variable ratio current transformer. Of course when very accurate measurements are required it is necessary to make the proper corrections in the indication of the instruments by obtaining the correct values from their calibration curves. To correct for errors due to instrument transformers see Article 67.

178. Arrangement of Meters.—The meters used in a given test must be so arranged that their indications are not influenced by stray magnetic fields. Iron vane instruments of the unshielded type may be made to indicate appreciable values when located near coils or conductors carrying currents even though there is no electrical connection to the terminals of the meters.

The best way to prepare for a given test is to first locate the meters in a respectable place where they may be easily read and then make the necessary electrical connections. In other words, make the connecting leads suit the location of the meters and do not make the location of the meter suit the length of wire which happens to be the next piece.

When the coils of a meter are properly shielded (enclosed in an iron shell) less precaution is necessary in the relative locatio of the meters, but precautions must be taken just the same, for is not always possible to tell from the outside appearance of th meter whether it is properly shielded or not.

179. Iron Vane Type of Meter.—The essential parts of th type of meter are shown in Fig. 158. An iron vane attached t the indicating element (the meter is located in tl magnetic axis of a fixe coil. When no curren flows through the coil tl iron vane assumes a pos tion as shown in the figur When current flows throug the coil two opposed torqu act on the indicating el ment. One of these torques is due to the spring, and tl other is due to the magnetic forces acting on the vane. Tl pointer assumes a final position where the driving torque and tl opposing torque are equal. This is a good type of inexpensi meter, but if unshielded it is very susceptible to the influences of stray magnetic fields.

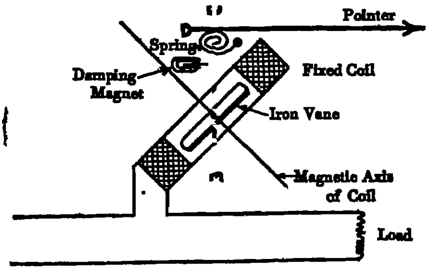


Fig. 158.—Iron Vane Type of Meter

180. Electrodynamometer Type of Wattmeter.—In this type of meter two coils are used. One of the coils is fixed in position and the other is mounted on the shaft of the indicating element. The movable coil is located in the magnetic field of the fixed coil. The meter shown in Fig. 159 is of this type. In this particul instrument the fixed coil is designed to carry the total line currer The movable coil is wound with fine wire and is connected series with a high resistance (located in the same case as t meter) directly across the line.

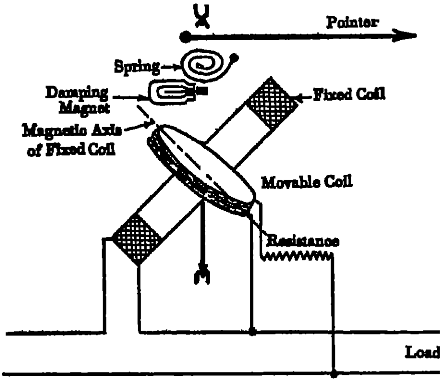


Fig. 159.—Electrodynamometer Type of Wattmeter

181. Induction Ammeter.—In this type of meter the torque acting on the indicating element is produced in the same way as the torque acting on the rotor of an ordinary polyphase induction motor. Thus in Fig. 160, which represents an induction ammeter, the two alternating fields ϕ_M and ϕ_B produce a revolving magnetic field which exerts a torque on the indicating element.

The flux ϕ_M is produced by the current in winding *A* and corresponds to the mutual flux in an ordinary power transformer, while the flux ϕ_B may be considered as a part of the secondary winding leakage flux. Due to the type of magnetic circuit the mutual flux is almost directly proportional to the current in *A*. The e.m.f. induced in *B* due to the mutual flux lags the mutual flux by 90 degrees and the current set up in this winding lags the e.m.f. by an angle which is determined by the constants of the winding. The flux ϕ_B is nearly in phase with the current in *B*. It follows that the time phase difference between the two fields depends on the constants of winding *B* and may be made roughly 90 degrees. Since the two fields are 90 electrical space degrees apart and are not in time phase it follows that a revolving mag-

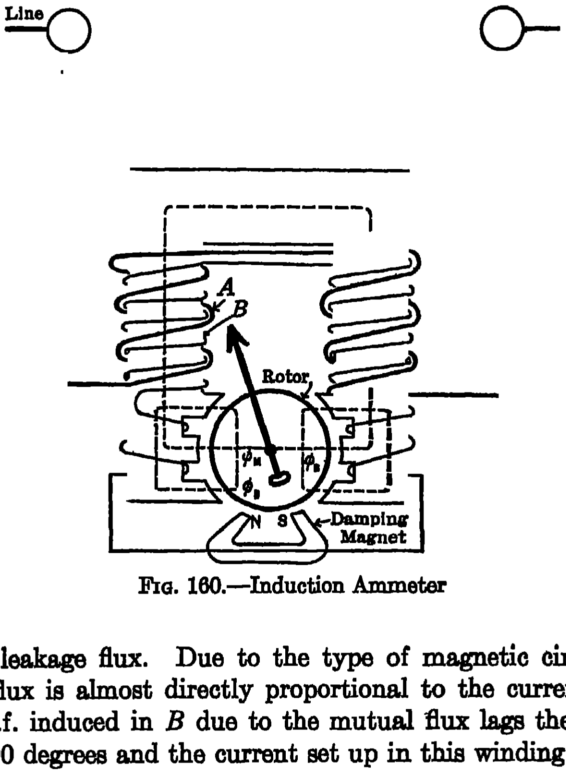


FIG. 160.—Induction Ammeter

netic field is produced. The torque on the indicating element depends on the product of the revolving field and the rotor reaction field and since both of these fields are proportional to the current in *A* it follows that the torque exerted on the indicating element of an induction ammeter depends on the square of the line current.

182. Induction Wattmeter (Fig. 161).—The magnetic circuit and indicating element of this meter are similar to those in an induction ammeter. Three coils are required. The potential coil *A* has a high impedance and is connected in series with a reactor. Coil *C* is designed to carry an appreciable current and corresponds to the current coil in an ordinary wattmeter. The auxiliary coil *B* is required to obtain the proper time phase relation between the two fields ϕ_M and ϕ_C.

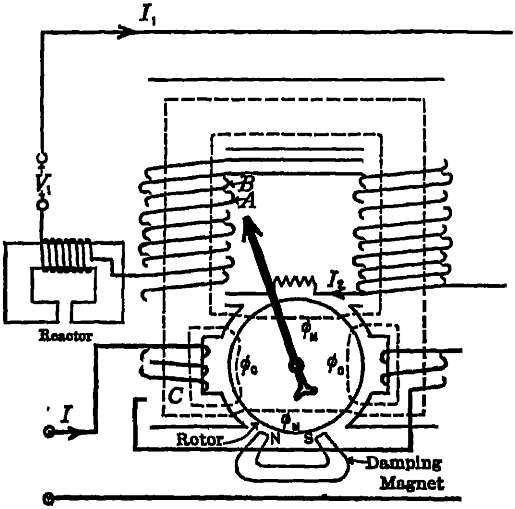


FIG. 161.—Induction Wattmeter

The flux ϕ_M represents the mutual flux of coils *A* and *B* and is determined by the current in *A*; that is, it depends on the voltage impressed on the potential circuit. The flux ϕ_C is proportional to the current in *C* and is therefore proportional to the line current in the circuit of which the power is measured.

To obtain correct meter indications when measuring the power of a unity power factor load it is necessary that the fluxes ϕ_M and ϕ_C be 90 degrees out of phase—consult a book on "Electrical

Meters." The flux ϕ_C is in phase with the line current and is therefore in phase with the line voltage at unity power factor. The position of ϕ_M with respect to the impressed voltage on the potential circuit depends on the power factor of the short-circuited winding (B). By adjusting the constants of B (see Fig. 163) it is possible to make the impressed voltage (line voltage) and the mutual flux (ϕ_M) 90 degrees out of phase. If the adjustment outlined is properly made the two fields ϕ_M and ϕ_C produce a torque on the rotor which is proportional to the true power in the circuit. (The reader should review the theory of the transformer.)

183. Induction Watt-hour Meter.—The principle of operation of this type of meter is similar to that of the induction wattmeter, but in this meter the indicating element rotates continuously at a slow speed and registers the power over a definite interval of time. Four coils are required (see Fig. 162). Coils A, B, and C perform the same function as the corresponding coils in the induction wattmeter and coil D (a one-turn coil) is required to compensate for the friction in the bearings and gearing of the indicating element.

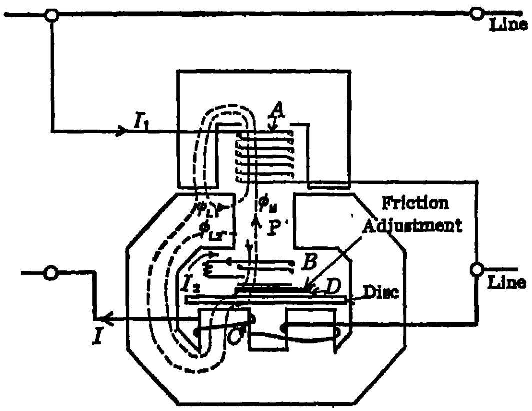


Fig. 162.—Induction Watthour Meter

Coils A and C produce magnetic fields whose resultant shifts in one direction across a circular aluminum disc which is attached to the main shaft of the indicating element. A torque results which turns the disc in a definite direction. The speed of the disc is retarded by permanent magnets (drag magnets) whose magnetic fields cut across the disc.

The driving torque exerted on the indicating element at any instant is proportional to the power at that instant provided, the resultant flux ϕ_2 across the disc due to the coils A and B and the impressed voltage on the potential circuit are 90 electrical time degrees apart—consult a book on "Electrical Meters." The proper phase relation may be obtained by adjusting the resistance (lag adjustment) of coil B. Fig. 163 shows how the proper phase relation may be obtained. The current I_{21} depends on the impedance of winding B and I_1 depends on I_{21}. When the proper value of I_{21} is obtained the leakage impedance drop of the potential circuit combined with the counter-voltage drop E_1 results in a voltage which is 90 degrees ahead of the useful flux ϕ_2.

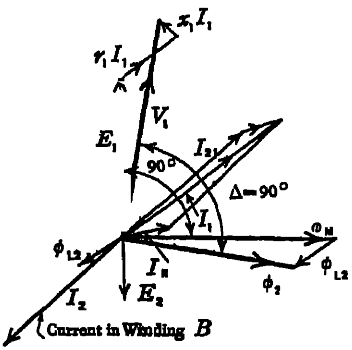


FIG. 163.—Vector Diagram Showing How Lag Adjustment May be Made

The friction adjustment is made by moving coil D across the face of the potential lug (P) until the disc is just on the point of turning in the positive direction when the current in coil C is zero and that in A has its normal value. The principle of operation of this adjustment is the same as that for the shaded pole single-phase induction motor; the displaced coil causes a magnetic field to shift from the unshaded to the shaded portion of the potential lug and thereby a slight torque is exerted on the aluminum disc.

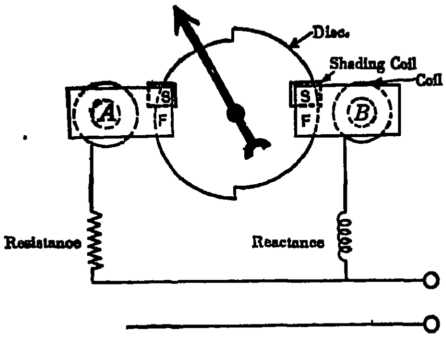


FIG. 164.—Induction Frequency Meter

184. Induction Frequency Meter.—The main parts of this instrument are shown in Fig. 164. Two shaded-pole electromagnets are required. One of the electromagnets is connected in series with a high resistance directly across the line and the other is supplied through an inductive reactance from the same line. The magnetic field of each electromagnet shifts from the unshaded to the shaded area and cuts the disc of the indicating element. Two opposed torques are produced. The disc is so shaped that when the torque due to one of the electromagnets is decreased the disc moves to a new point where the respective torques balance. If for a given impressed voltage the frequency is higher than normal, the current in electromagnet *A* will be relatively higher than that in *B* and the torque due to *A* will be greater than that due to *B* with the result that the disc moves to a new location where less area of the disc will be under the influence of *A*. If the frequency is lower than normal the disc moves to a new location in the opposite direction.

185. Vibrating Reed Frequency Meter.—One common type of frequency meter consists of a bank of steel reeds each one of which is tuned to mechanically vibrate at a given frequency (see Fig. 165). The mechanical vibration is set up by an electromagnet *E* which is polarized by a permanent magnet *M*. The electromagnet is connected in series with a high resistance to the source of which the frequency is desired. On account of the permanent magnet, every positive half of the current in the electromagnet results in a mechanical impact on all the reeds in the instrument with the result that the reed which is mechanically tuned to the frequency of the alternating-current vibrates with the greatest amplitude.

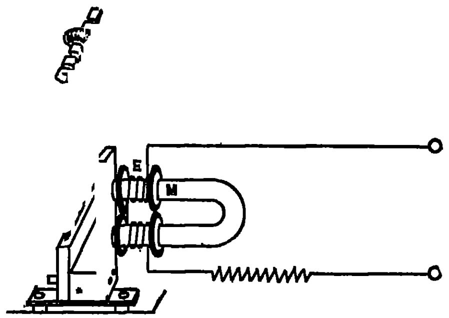


FIG. 165.—Vibrating Reed Frequency Meter

186. Synchroscope.—See Article 97.

187. The Thomson Resistance Bridge.—When it is necessary to know the ohmic resistances of the windings on electrical machines it is sometimes necessary to make measurements of resistances which have a very low value. Probably the best way to make such measurements is to use a Thomson Bridge (see Fig. 166). The ratio arms *AA* and *BB* are so adjusted that a small change in the known resistance *K* results in a radical change he galvanometer deflection. The current through the circuit uld be as high as the bridge permits, but it should not be so n as to change the value of the resistance X while the necessary

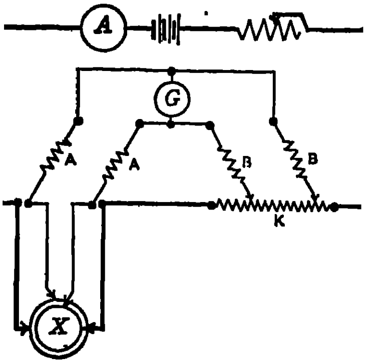


FIG. 166.—Thomson Bridge

ustments are made. The values of A, B, and K are observed en the galvanometer indicates zero and the unknown resistance y be calculated by the relation

$$X=\frac{A}{B}K$$

CHAPTER XX

MACHINERY LABORATORY PROBLEMS

1. Obtain, using an oscillograph some of the following wave shapes:

(*a*) Voltage across an air-core reactor and the current through the reactor.

(*b*) Voltage across an iron-core reactor and the current through the reactor.

(*c*) Voltage across a condenser and the current through the condenser.

(*d*) Impressed voltage, anode current, and load current of a rectifier.

(*e*) Output current of a d-c. magneto.

(*f*) Voltage and current of an a-c. arc lamp.

(*g*) Terminal voltage, field current, and armature current of a single-phase alternator.

(*h*) Repeat (*g*), using a polyphase generator connected to a balanced load.

Give reasons for the shapes of the waves obtained in each case.

2. Obtain the wave shapes for the impressed voltage and the no-load current of a power transformer.

Give reasons for the shape of the exciting current wave.

Analyze the wave of current for the fundamental and all harmonics up to and including the 5th.

Why is it possible to represent the no-load current of a transformer by a single vector of constant magnitude on a vector diagram?

3. Connect the secondary winding of a power transformer in series with lamps and a high inductive reactance to a source of direct-current. (The purpose in doing this is to obtain magnetism, corresponding to residual magnetism, in the core.) Connect the primary winding of the transformer to a source of alternating-current and obtain the wave shape of the primary current.

Give reasons for the shape of the current wave.

Analyze the wave for the even harmonics.

4. Determine the total inductance of an air-core reactor. Make several measurements using the same frequencies but radically different values of current.

Explain why the values for inductance do or do not agree.

What factors influence the accuracy of your measurements?

5. Determine the total inductance of an iron-core reactor. (One of the windings of a power transformer may be used.) Make several measurements using the same frequencies but radically different values of current.

Explain why the values for inductance do or do not agree.

6. For a given frequency measure the power input and the power factor of a leaky condenser.

Calculate the value of a pure resistance and a pure capacity which when connected in series are equivalent to the leaky condenser at the given frequency.

Calculate the value of a pure resistance and a pure capacity which when connected in parallel are equivalent to the leaky condenser.

7. Connect an air-core reactor and a condenser in series to an a-c. supply. (The current through the circuit must not exceed a value furnished by the instructor.) For an approximately constant impressed voltage determine a curve showing the relation between impedance and frequency. (The voltage across certain parts of the circuit may be very high and great care must be taken in making the measurements.)

What is the significance of the impedance curve?

Construct a complete voltage drop-current vector diagram for the condition of minimum impedance in the circuit.

8. Connect an air-core reactor and a condenser in parallel to an a-c. supply. Determine a curve showing the relation between impedance and frequency.

What is the significance of the curve?

Construct a complete voltage drop-current vector diagram of the circuit for the frequency at which the impedance is a maximum.

9. Given a circuit of unknown constants locked in a box. Determine by measurements, the values of a pure resistance, pure inductance, and pure capacity which, when properly connected, form a circuit approximately equivalent to the one in the box.

What errors are likely to be made in the measurements of the constants and the form of the circuit?

10. Determine by means of a phase meter or an oscillograph the voltage relations in a balanced Y-connected load. Prove by using the same instruments that in a balanced three-phase unity power factor load the line current and the line voltage are not in phase.

Explain in your report how these phase relations were actually measured.

11. By means of a phase meter or an oscillograph determine the relative phase rotation of two independent three-phase systems operating at the same frequency.

12. Measure the power and the power factor of a three-phase induction motor at no-load and at full-load. In making the measurements use the following methods:

(*a*) Three wattmeter method.

(*b*) Two wattmeter method.

(*c*) Single wattmeter method with a Y-box.

(All meters should be connected in the circuit in such a way that the three methods may be checked simultaneously.)

Compare the results and explain why they do or do not check.

13. (*A*) In the two wattmeter method for measuring power determine experimentally whether the meter indications must be added or subtracted.

(*B*) For a given load note the effect on the indication of the two wattmeters when the phase rotation of the generator voltages is changed.

(*C*) By means of an oscillograph or a phase meter determine the phase relations between the current and voltage of each wattmeter.

Draw complete voltage-current vector diagrams for parts (*A*) and (*B*). Explain fully what assumptions you made.

14. Note the effects of stray magnetic fields on the indications of various a-c. meters. The stray fields may be produced by a coil which is excited in the following ways:

(*a*) Direct-current.

(*b*) Alternating-current having the same frequency as that of the current in the circuit to which the meter is connected.

(*c*) Alternating-current of slightly different frequency than that in (*b*).

(*d*) Alternating-current having radically different frequency than that used in (*b*).

(In making the observations the meters should be located in a place where the stray field has the maximum effect.)

Explain the results obtained.

15. Connect an air-core reactor in series with a variable resistor to a constant voltage, constant frequency supply. Obtain the locus of the current when the resistance is varied from zero to a high value.

The curve obtained should be a circle whose diameter lags the voltage by 90 time degrees.

Give reasons for the shape of the actual curve obtained.

16. Connect a resistor in series with a variable air-core reactor to a constant voltage, constant frequency supply. Obtain the locus of the current when the resistance of the circuit is constant and the reactance varied from zero to high value.

The curve obtained should be a circle whose diameter is in phase with the voltage.

Give reasons for the shape of the actual curve obtained.

17. Determine by direct test the voltage ratio and the approximate turn ratio of a power transformer.

See A. I. E. E. Standardization Rules in regard to these ratios.

18. Given a power transformer. Measure:

(*a*) Self-inductance of windings.

(*b*) Mutual inductance between the primary and the secondary windings.

(*c*) Turn ratio.

Calculate the leakage inductances of the primary and secondary windings.

What errors are likely to be made in determining the inductance of a winding?

19. At constant frequency determine curves showing the relations between impressed voltage and no-load current and the impressed voltage and core-loss of a power transformer. The voltage must be varied over wide limits including points well above and below the rated voltage.

Give reasons for the shapes of the curves obtained.

20. At constant voltage determine curves showing the relation between impressed frequency and no-load current and between impressed frequency and core-loss of a transformer. The frequency must be varied over a wide range.

Give reasons for the shapes of the curves.

21. Determine by test the polarities of all the windings on a transformer.

See A. I. E. E. Rules in regard to the proper way for marking the leads of a transformer.

22. Determine by actual load tests (record temperatures) the voltage regulation of a power transformer delivering power to the following loads:

(*a*) Unity power factor rated kv-a. load.
(*b*) Highly inductive rated load.
(*c*) Highly condensive rated load.

Since the voltage regulation of a power transformer is usually small, great care must be taken in measuring the voltages.

Explain your results in detail.

23. Determine by actual load tests (record temperatures) efficiency-load curves for a transformer operated at rated secondary voltage and frequency and supplying the following loads:

(*a*) Approximately unity power factor load.
(*b*) A 50 per cent lagging power factor load.

It may be advisable to use a power factor meter in part (*b*).

24. Measure the no-load and the load losses (record temperatures) of the transformer specified in Problem 23. See A. I. E. E. Standardization Rules in regard to the method for measuring these losses. (Note that low range ammeters and current coils of wattmeters must be protected by a short-circuiting switch.)

On the basis that input = output + losses calculate the efficiency of this transformer for rated unity power factor load. Assume that the operating temperature of the transformer is the same as that measured in Problem 23.

25. (*A*) Short-circuit the low-voltage winding of a transformer and measure its impedance on the high-voltage side. Obtain the impedance for many values of current ranging between ½ to 5 times the rated current of the high-voltage winding. The impedances must be measured at rated frequency.

Plot the following curves:

(*a*) Equivalent resistance against current input.
(*b*) Equivalent leakage reactance against current.
(*c*) Equivalent impedance against current.

(*B*) Short-circuit the high-voltage winding and measure the equivalent impedance and equivalent resistance of the transformer referred to its low-voltage winding. Plot curves similar to those in (*a*), (*b*), and (*c*), of part (*A*).

(*C*) Measure the turn ratio of the transformer.

Calculate the following ratios and compare them with the turn ratio of the transformer:

(*a*) The ratio of the equivalent impedances measured on the high-voltage and the low-voltage sides of the transformer.
(*b*) The ratio of the equivalent resistances.
(*c*) The ratio of the equivalent leakage reactances.

Explain your results.

26. (*A*) Determine by measurements the constants of the exactly equivalent circuit of a transformer referred to its low-voltage winding. (Record temperatures.)

(*B*) Measure the voltage required across the high-voltage winding, necessary to obtain rated voltage across the low-voltage winding when the low-voltage winding delivers rated current to a non-reactive load. (Record temperatures.)

Using the exactly equivalent circuit obtained in part (*A*) calculate the voltage required across the high-voltage winding of the transformer when the low-voltage winding delivers rated current at rated voltage to a non-reactive load. Compare these results with those obtained in (*B*). (Note the constants of the circuit must be referred to the temperature obtained in (*B*)).

(*C*) Determine the constants of the approximately equivalent circuit for the transformer referred to the low-voltage winding. On the basis of this circuit calculate the voltage required across the high-voltage winding when the low-voltage winding is operated under the same conditions of load as specified above.

Explain your results.

27. Using the approximately equivalent circuit determined in Problem 26, calculate the voltage regulation of the transformer by the A. I. E. E. method for:

(*a*) Unity power factor rated kv-a. load.

(*b*) An 80 per cent lagging power factor rated kv-a. load.

(*c*) An 80 per cent leading power factor rated kv-a. load.

28. Determine the magnitude of the error made due to the phase angles of instrument transformers in measuring the power and power factor of the following loads requiring approximately 100 watts each:

(*a*) Highly inductive load.

(*b*) Highly condensive load.

(*c*) Unity power factor load.

29. The all-day load requirements on a transformer are as follows:

0.5 kilowatt output for 4 hours;
1.0 kilowatt output for 11 hours;
3.0 kilowatt output for 1 hour;
and 0.0 kilowatt output for 8 hours.

Transformers No. —— or No. —— may be used to supply the load. Determine by short-cut methods of test which one of the two transformers will give the best operation.

Explain in detail your method of determining the proper transformer for supplying this load.

What assumptions did you make?

30. By means of two transformers properly wound for the T-connection (Scott connection), transform two-phase to three-phase power (or vice versa) The three-phase load should be nearly balanced. Measure the currents, voltages, and power factors of the circuits.

Construct a complete voltage-current vector diagram for the primary and secondary windings of the transformers.

31. Connect two transformers in open-delta to a balanced three-phase supply and a balanced three-phase load. Measure the voltage and currents in the windings and determine their phase relations.

Construct a voltage-current vector diagram showing the relations between the voltages and the currents in the windings.

Calculate the power output of each transformer.

32. Transform from three-phase to six-phase by the following methods:

(*a*) Three transformers connected in double delta.

(*b*) Three transformers connected in double Y.

(*c*) Three transformers connected diametrically.

Give diagrams showing how the six-phase voltages are obtained in each case.

33. (*A*) Determine by short-cut methods of test whether it is possible to operate transformers No. —— and No. —— in parallel.

(*B*) Actually operate the transformers in parallel and measure the voltage, current and power of each winding for different values of load.

Explain the method used in (*A*).

Explain the results obtained in (*B*).

How is it possible to make these transformers share the load properly?

34. Adjust the field current of a single-phase a-c. generator so as to obtain at no-load and at rated speed, 125 per cent rated name plate voltage. With the field current and the speed constant obtain curves showing the relation between terminal voltage and armature current for:

(*a*) Unity power factor loads.

(*b*) Eighty per cent lagging power factor loads.

(*c*) Eighty per cent leading power factor loads.

A phase or power factor meter should be used in determining the power factor of the load.

Give reasons for the shapes of the curves obtained.

35. Obtain by simultaneous measurements of the input and useful output of an a-c. generator its efficiency at rated armature current, rated voltage, and rated speed for:

(*a*) Unity power factor load.

(*b*) Eighty per cent lagging power factor load.

(*c*) Eighty per cent leading power factor load.

Measure the resistance and temperature of the armature and field windings.

In this test, assume that the rated current is independent of the power factor of the load and that it is equal to the current calculated from the name plate rating.

36. Determine the conventional efficiency of the alternator in Problem 35 for the loads specified in the problem. Before an attempt is made to do this test, the student must read the A. I. E. E. Standardization Rules in regard to the method for determining the conventional efficiency. The following quantities are required:

(*a*) Field and armature resistances.

(*b*) A curve showing the relation between core-loss and speed at constant rated field current. [The rated field current is the current required to give rated armature terminal voltage + RI (armature resistance drop at rated armature current) at no-load and at rated speed.]

(*c*) A curve showing the relation between friction (bearing, brush, and windage) and speed.

(*d*) A curve showing relation between stray-load losses and armature current at rated speed.

37. In this investigation a three-phase armature winding having (six) terminals brought out to a terminal board is required. The stator of a three-phase motor with its rotor removed may be used.

(*a*) Determine the location and the number of magnet poles formed by the armature current when phase 1 is excited by 10 amperes d-c. Mark the terminal through which the current enters the phase positive.

(*b*) Repeat (*a*) when phase 2 is excited by d-c.

(*c*) Repeat (*a*) when phase 3 is excited by d-c.

(In a three-phase machine when the phases are excited by direct-current and in the proper sequence the magnet poles advance around the armature in a certain direction and a distance of 120 electrical space degrees for each change in excitation.)

(*d*) Using the results obtained in parts (*a*), (*b*), and (*c*), determine the proper way for connecting the windings in Y and delta.

(*e*) Connect the phases in Y and connect the terminals to a balanced three-phase supply. Explore the magnetic field.

(*f*) Repeat (*e*) with the phases connected in delta.

(*g*) Reverse the phase rotation of the exciting source and note the effect on the magnetic field.

38. Make a thorough study of the A. I. E. E. Standardization Rules in regard to the methods for determining the voltage regulation of alternators.

(*A*) Determine by actual test the open-circuit saturation curve and the zero power-factor full-load saturation curve for a polyphase alternator.

(*B*) Determine a curve showing the relation between the armature short-circuit current (all phases must be short-circuited) and the field current.

(*C*) Determine by direct test and by the A. I. E. E. method the voltage regulation of the alternator for the following loads:

- (*a*) Unity power factor rated kv-a. load.
- (*b*) Ninety per cent lagging power factor rated kv-a. load.
- (*c*) Ninety per cent leading power factor rated kv-a. load.

39. (*A*) Determine with synchronizing lamps the relative phase rotations of two synchronous machines which are to be operated in parallel.

(*B*) By actual tests familiarize yourselves with the following methods of synchronizing polyphase synchronous machines:

- (*a*) Synchronizing lamps so connected that they are dark when exact synchronism is obtained.
- (*c*) Synchroscope.
- (*d*) Induction motor action.

Give a thorough explanation of each method.

40. (*A*) Operate an alternator in parallel with the laboratory source of a-c. power. It is assumed in this problem that the alternator is driven by an ordinary d-c. motor and that the voltage and frequency of the a-c. power are determined by a large generator supplying the laboratory. Obtain readings for armature current, field current, and voltage of the motor and the generator for the following loads on the alternator:

(*a*) Full-load current at unity power factor.

(*b*) Full-load current at 70 per cent lagging power factor.

(*c*) Full-load current at 70 per cent leading power factor.

(*B*) For a given field current (say the field current in (*a*)) of the alternator obtain a curve showing the relation between the alternator output and the field current of the driving motor.

(*C*) For a given field current (say the field current in (*a*)) of the driving motor obtain a curve showing the relation between the power factor of the alternator and the field current of the alternator.

Give reasons for the results obtained.

41. Load two alternators in parallel. Maintain constant rated voltage and frequency and obtain meter readings (field current, armature current, and voltage of each driving motor and alternator; power output of each alternator, and the total output; and frequency) for the following loads:

(*a*) Full non-reactive load current on each alternator.

(*b*) Full non-reactive load current on one alternator, and zero load current on the other.

(*c*) Full 80 per cent leading power factor load current on one alternator and full-load current on the other. (A non-reactive load must be used and the proper power factors must be obtained by adjustments of the field currents.)

Explain in detail the results of the test.

42. Operate an a-c. generator at rated speed. Adjust the generator field current so as to obtain 115 per cent rated terminal voltage at no-load and keep the field current constant throughout the test. Determine by actual measurements the maximum 50 per cent lagging power factor (watt) load which may be supplied by the generator without exceeding the rated current of the armature of the generator. (It is assumed in this test that apparatus is available for power factor correction.)

In your report explain in detail the results obtained. Explain your method for determining the maximum load.

43. Determine the various losses in an alternator using the retardation method. (See "Karapetoff's Experimental Electrical Engineering" Sections 329–337.)

44. Determine by various kinds of measurements the voltage, current, and power rating of an alternator. (See A. I. E. E. Rules in regard to the rating of a-c. generators.)

Explain your method for determining the rating of the machine.

45. (*A*) Obtain V-curves for a synchronous motor at 0, ¼, ½, and full-load. Operate the motor at rated voltage and frequency. In determining the points for a curve, keep the power output of the motor constant.

From the V-curves obtain the compounding curve for unity power factor.

(*B*) Make the necessary measurements for predetermining by calculations the points on the V-curves.

Using the results obtained in (*B*) calculate the field current required on the motor to obtain unity power factor at rated load. Compare this current with that calculated in part (*A*).

46. (*A*) Determine by test the reactive kv-a. condensive load which must be connected in parallel with a —— kilowatt highly lagging power factor load to obtain a resultant load having 90 per cent lagging power factor.

(*B*) Determine the most economical way for making the power factor correction in part (*A*).

(*C*) Determine by test the reactive kv-a. condensive load required in part (*A*) to correct the power factor from 90 per cent lagging to unity.

Under what conditions does it pay to correct the power factor of a circuit from 90 per cent lagging to unity?

47. (*A*) Study by actual tests various methods for putting synchronous converters into operation.

(*B*) Start a synchronous converter by induction motor action and obtain the correct d-c. polarity by "slipping a pole."

Explain in detail the performance of the converter in part (*B*).

48. (*A*) Drive a synchronous converter from a d-c. source and measure all possible voltage ratios.

Compare the measured voltage ratios with the theoretical ratios.

Give a wiring diagram showing how the armature of the converter is connected to the slip rings and the commutator.

(*B*) Drive the converter from the a-c. end. Keep the impressed a-c. voltage and frequency constant. Adjust the field current so as to obtain unity power factor for all loads. Obtain the following curves:

(*a*) Efficiency plotted against current output.
(*b*) Field current plotted against current output.

Explain the results obtained.

(*C*) Connect reactors in series with the a-c. leads and operate the converter in the usual way. Adjust the direct-current to ½ rated output (hold constant.) Obtain the following curves:

(*a*) Slip ring voltage plotted against field current.
(*b*) D-c. voltage plotted against field current.

Explain the results.

49. Obtain complete performance curves for a mercury vapor rectifier. Measure the voltages across the various parts of the rectifier.

Explain results.

50. Operate a polyphase induction motor and determine the following curves:

(*a*) No-load current plotted against impressed voltage and no-load power plotted against impressed voltage. (Note the frequency must be kept constant.)

(*b*) Efficiency, power factor, and slip curves plotted against horse power output. This test must be performed at rated voltage and frequency and with the rotor of the motor short-circuited.

(*c*) Repeat (*b*) with equal resistances connected externally in each rotor phase.

Explain the results in detail.

51. Make the necessary measurements required to construct the circle diagram for the motor tested in Problem 50. Construct the diagram and obtain from the diagram the curves specified in part (*b*), Problem 50.

52. Operate the motor used in Problem 50 on one phase and determine the characteristic curves. (All rotor phases must be short-circuited.)

Compare the curves with those obtained in Problem 50.

53. Obtain the following curves on the motor used in Problem 50.

(*A*) Starting torque plotted against impressed voltage at rated frequency. (Rotor short-circuited.)

(*B*) Starting torque at rated impressed voltage and frequency plotted against rotor resistance.

(*C*) Complete speed-torque curves at rated primary voltage and frequency for the following conditions:

- (*a*) Rotor short-circuited.
- (*b*) Rotor resistance of such value that the maximum torque is obtained at starting.
- (*c*) Rotor resistance too high for an appreciable starting torque.

54. Operate the motor in Problem 50 on one phase and with the rotor short-circuited. Obtain at rated voltage and frequency a complete speed-torque curve.

Compare this curve with that obtained for polyphase operation in Problem 53.

55. Drive a synchronous converter by an induction generator. (The best way to get the machines in operation is to run the converter inverted and start the induction generator as an induction motor from a-c. power supplied by the converter. The d-c. source to the converter may then be disconnected and the driving motor of the induction generator will keep the machines running.)

(*A*) Determine by actual test what factors determine the frequency of the current.

(*B*) Determine by actual test what factors determine the terminal voltage of the induction generator.

(*C*) Operate the induction generator at constant voltage and frequency. Load the d-c. end of the converter and determine the following curves:

- (*a*) Power factor of induction generator plotted against power output.
- (*b*) Slip of induction generator plotted against power output.

Explain results.

56. Connect a rotary phase advancer to the rotor of a polyphase induction motor. Operate the induction motor at rated voltage and frequency. For a constant mechanical load on the induction motor obtain a curve showing the relation between the power factor of the induction motor and the speed of the phase advancer.

Explain in detail the results obtained.

57. (*A*) Load one phase of a polyphase alternator. Adjust the voltage across the load to the rated terminal voltage of the alternator. Measure the voltages across all phases of the alternator and the line current.

(*B*) Operate a synchronous motor at no-load and in parallel with the load in (*A*). Adjust the excitation of the synchronous motor so as to obtain approximately unity power factor on the motor. With the same alternator field current and speed as in (*A*), measure the terminal voltages across all phases and the line currents.

Compare the voltages and currents obtained in (*A*) and (*B*).

Explain the balancing action of polyphase motors.

58. Obtain the characteristic curves for the following machines:

(*a*) Constant-current transformer.
(*b*) Single-phase repulsion motor.
(*c*) Single-phase series motor.
(*d*) Single-phase compensated induction motor.
(*e*) Synchronous-induction motor.
(*f*) Shunt polyphase commutator motor.

59. (*A*) Compare the indication of a commercial induction watthour meter with the indication of a rotating standard watthour meter for the following loads:

(*a*) Zero to 150 per cent non-reactive load.
(*b*) Zero to 150 per cent highly inductive load. (A power factor meter should be used in this test to assist in keeping power factor of the load constant.)

Plot the errors observed in (*a*) and (*b*) in terms of the volt-ampere load.

(*B*) Note the effect on the indication of the watthour meter when the following adjustments are changed:

(*a*) Friction adjustment.
(*b*) Drag magnet adjustment.
(*c*) Lag adjustment.

Explain results.

(In part (*B*) it is usually desirable to use a different meter than in part (*A*)).

60. Adjust an over-load induction type relay so that it operates as follows:

(*a*) Contacts close in —— seconds with —— amperes flowing through the relay winding.

(*b*) Contacts close in ——seconds with —— amperes flowing.

In part (*b*) obtain a curve showing the relation between the current and the time of closing.

(A cycle counter should be used in making the adjustments required in this problem.)

61. By means of an oscillograph obtain oscillograms of the following transients:

(*a*) Growth of current in an inductive circuit when the circuit is connected to a source of d-c. power.

(*b*) Decay of current in an inductive circuit.

(*c*) Current in a condensive circuit when the circuit is connected to a source of d-c. power.

(*d*) The exciting current of a power transformer at the instant when it is connected to a source of a-c. power.

(*e*) The primary and the secondary currents in a loosely coupled circuit in which the secondary winding is connected across a condenser and the primary winding across the terminals of a charged condenser.

(*f*) Current in a tungsten lamp when it is connected to a source of d-c. power.

(*g*) Current in a carbon lamp when it is connected to a source of d-c. power.

Explain your results.

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